

# Workbook



## Table of Contents

Vectors .....	3
Introduction to 2D and 3D Vectors.....	3
Vector Arithmetic.....	3
Vectors Dot Product.....	4
Vectors Cross Product.....	5

# Vectors

## Introduction to 2D and 3D Vectors

---

### Questions:

- 1) Find each of the following 2D/3D vectors, its magnitude, and whether it's a unit vector:
  - a. The displacement vector from  $(-8, 3)$  to  $(5, -2)$ .
  - b. The displacement vector from  $(2, 3, 4)$  to  $(2, 4, 4)$ .
  - c. The position vector for  $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ .
  - d. The position vector for  $(-8, 3, 5)$ .

The vector  $\vec{v} = \langle 7, -3, 0 \rangle$  starts at point  $P(-3, 4, -1)$ . At what point does it end?

## Vector Arithmetic

---

### Questions:

- 2) Given the 2D vectors  $\vec{a} = \langle 7, 4 \rangle$  and  $\vec{b} = \langle -2, 5 \rangle$  compute the following:
  - a.  $5\vec{a}$
  - b.  $6\vec{b} - 3\vec{a}$
  - c.  $\|9\vec{a} + 4\vec{b}\|$
  
- 3) Given the 3D vectors  $\vec{u} = 7\vec{i} - 2\vec{j} + 4\vec{k}$  and  $\vec{v} = 6\vec{j} - 2\vec{k}$  compute the following:
  - a.  $-4\vec{v}$
  - b.  $10\vec{u} + \vec{v}$
  - c.  $\|-8\vec{u} - 3\vec{v}\|$

4) Answer the following questions:

- Find a unit vector that points in the same direction as  $\vec{v} = \vec{i} - 4\vec{j} + 8\vec{k}$ .
- Find a vector that points in the same direction as  $\vec{w} = \langle -2, 5 \rangle$  with a magnitude of 10.

5) Determine if the following pairs of vectors are parallel.

- $\vec{v} = 6\vec{i} - 4\vec{j} - 16\vec{k}$  and  $\vec{w} = 15\vec{i} - 10\vec{j} - 40\vec{k}$ .
- $\vec{a} = \langle 3, -2, 5 \rangle$  and  $\vec{b} = \langle 6, -4, 7 \rangle$ .

## Vectors Dot Product

---

6) In each of the following, find the dot product  $\vec{a} \cdot \vec{b}$ :

- $\vec{a} = \langle 5, -4 \rangle$  and  $\vec{b} = \langle 4, 3 \rangle$ .
- $\vec{a} = 8\vec{i} + 6\vec{j} - 3\vec{k}$  and  $\vec{b} = 6\vec{i} - 4\vec{j} + 7\vec{k}$ .
- $\|\vec{a}\| = 4$ ,  $\|\vec{b}\| = 3$ , and the angle between the two vectors is  $\frac{\pi}{3}$ .

7) In each of the following, find the angle between the two vectors:

- $\vec{a} = \langle 3, 5 \rangle$ ,  $\vec{b} = \langle 7, 6 \rangle$ .
- $\vec{v} = \vec{i} - 2\vec{j} + 3\vec{k}$ ,  $\vec{w} = 5\vec{i} + 6\vec{j} - 7\vec{k}$ .

8) In each of the following, determine if the two vectors are parallel, orthogonal or neither:

- $\vec{p} = \langle 1, -2, 3 \rangle$ ,  $\vec{q} = \langle 5, -8, -7 \rangle$ .
- $\vec{a} = \langle 3, 5 \rangle$ ,  $\vec{b} = \langle 7, 6 \rangle$ .
- $\vec{v} = \vec{i} - 2\vec{j} + 3\vec{k}$ ,  $\vec{w} = -5\vec{i} + 10\vec{j} - 15\vec{k}$ .

9) Compute  $proj_{\vec{u}} \vec{v}$  [projection of  $\vec{v}$  onto  $\vec{u}$ ] for the following pairs of 2D/3D vectors:

- $\vec{u} = \langle 4, -1 \rangle$ ,  $\vec{v} = \langle 1, 7 \rangle$ .
- $\vec{u} = 7\vec{i} - \vec{j} + \vec{k}$ ,  $\vec{v} = -2\vec{i} + 5\vec{j} - 6\vec{k}$ .

10) Find the direction cosines and direction angles for  $\vec{v} = \vec{i} - 2\vec{j} + 3\vec{k}$ .

## Vectors Cross Product

---

### Questions:

11) Answer the following questions:

a. Given  $\vec{a} = \langle 3, -2, 5 \rangle$  and  $\vec{b} = \langle 6, -4, 7 \rangle$ .

Compute  $\vec{a} \times \vec{b}$  and use the result to find  $\vec{b} \times \vec{a}$ .

b. Given  $\vec{u} = 3\vec{i} - \vec{j} + 5\vec{k}$  and  $\vec{v} = 4\vec{j} - 2\vec{k}$ .

Compute  $\vec{u} \times \vec{v}$  and use the result to find  $\vec{v} \times \vec{u}$ .

12) Find a vector that is orthogonal to the plane containing the points  $P(1, 2, 3)$ ,  $Q(6, 5, 4)$  and  $R(7, 8, 9)$ .

13) For each of the following sets of 3 vectors, determine if they lie in the same plane or not:

a.  $\vec{a} = \langle 3, -2, 5 \rangle$ ,  $\vec{b} = \langle 6, -4, 7 \rangle$ ,  $\vec{c} = \langle 1, 0, 1 \rangle$ .

b.  $\vec{u} = \vec{i} + 4\vec{j} - 7\vec{k}$ ,  $\vec{v} = 2\vec{i} - \vec{j} + 4\vec{k}$ ,  $\vec{w} = -9\vec{j} + 18\vec{k}$ .

**Answer Key:**

- 1) a.  $\langle 13, -5 \rangle$       b.  $\langle 0, 1, 0 \rangle$       c.  $\left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$       d.  $\langle -8, 3, 5 \rangle$  ;  $(4, 1, -1)$
- 2) a.  $\langle 35, 20 \rangle$       b.  $\langle -33, 18 \rangle$       c.  $\sqrt{6161}$
- 3) a.  $-24\vec{j} + 8\vec{k}$       b.  $70\vec{i} - 14\vec{j} + 38\vec{k}$       c.  $\sqrt{3816}$
- 4) a.  $\frac{1}{9}\vec{v} - \frac{4}{9}\vec{j} + \frac{8}{9}\vec{k}$       b.  $\left\langle \frac{-20}{\sqrt{29}}, \frac{50}{\sqrt{29}} \right\rangle$
- 5) a. Parallel      b. Not parallel
- 6) a. 8      b. 3      c. 6
- 7) a.  $18.435^\circ$       b.  $44.479^\circ$
- 8) a. orthogonal      b. neither      c. parallel
- 9) a.  $\left\langle \frac{-12}{17}, \frac{3}{17} \right\rangle$       b.  $\left\langle \frac{-161}{51}, \frac{23}{51}, \frac{-23}{51} \right\rangle$
- 10)  $\sqrt{14}$
- 11) Part 1:  $\langle 6, 9, 0 \rangle$       Part 2:  $18\vec{i} - 6\vec{j} + 12\vec{k}$
- 12)  $\langle 12, -24, 12 \rangle$
- 13) a. Not in the same plane      b. In the same plane.