

# Workbook



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# The Derivative of a Function

## Basic Derivatives of Functions

### Questions:

1) Find the first derivative:

a. $f(x) = 4$	b. $g(x) = \frac{e + \sqrt{2}}{2}$	c. $h(x) = x^4$
d. $y = \frac{1}{x^2}$	e. $f(x) = \sqrt{x}$	f. $y = \frac{1}{\sqrt{x}}$
g. $y = 4x^{10} + \frac{1}{x}$	h. $y = x + x^2$	i. $y = 4x^2 + 8x^3 - 5$
j. $y(x) = \frac{2}{x} - ex$	k. $y = \sqrt{2}x^2 + 2ex$	l. $y(t) = \frac{4}{t} + \sqrt[3]{t}$

2) Find the first derivative:

a. $y = (x^2 + 3)(x - 1)$	b. $y = (4x + 10)(\sqrt{x} - 1)$
c. $y = (x - 1)(x - 1)(x - 2)$	d. $y = \frac{4x + 10}{x^2 - x}$
e. $y = \frac{x^2 + 4x - 1}{2x - 3}$	f. $y = \frac{ex + 1}{ex - 1}$
g. $y = (4x + 10)^3$	h. $y = (x^2 + 1)^5$
i. $y = \sqrt{(x^2 + x + 1)^3}$	j. $y = (2x + 1)^3(4x - 5)^4$
k. $y = \frac{(2x + 3)^4}{(x - 5)^3}$	l. $y = \frac{1}{\sqrt[3]{4x + 1}}$

## Derivative of Exponents and Logarithmic Functions

### Questions:

3) Find the first derivative:

a.  $y = e^x$

b.  $y = 4e^x + 2x^3$

c.  $y = e^x(x^2 + x + 4)$

d.  $y = \frac{e^x}{x^2 - x}$

e.  $y = e^{4x-1} + e^{2x}$

f.  $y = e^{-x}(x+1)$

g.  $y = \frac{e^{2x} - x}{e^{4+x}}$

h.  $y = e^{\sqrt{x}}$

i.  $y = \frac{e^{\pi x}}{x-2}$

j.  $y = \frac{1}{\sqrt{e^{4x} + 1}}$

k.  $y = \sqrt[3]{e^{x^2+1} + 1}$

l.  $y = \frac{e^{-x^2}}{x}$

4) Find the first derivative and second derivatives:

a.  $y = \frac{\ln x}{x}$

b.  $y = \ln^2 x + \frac{1}{\ln x}$

c.  $y = e^{2x} \ln(x^2 + 4)$

d.  $y = \frac{\ln x}{e^x}$

## Trigonometric Derivatives

### Questions:

5) Find the first derivative:

a.  $f(x) = \sin x$

b.  $g(x) = \sin 4x$

c.  $y = \cos(0.5x)$

d.  $y = \sin^2 x$

e.  $f(x) = \cos^4(5x)$

f.  $z(x) = \sqrt{\sin 2x}$

g.  $y = \sin x \cos 3x$

h.  $y = \frac{\sin x - 1}{\cos 2x + 2}$

i.  $y = x^3 \sin 4x$

j.  $y(x) = \ln(\cos(x))$

k.  $y = e^{\sin 2x} \ln x$

l.  $y(t) = \sin(\cos(x))$

## Derivative of Power Functions

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### Questions:

6) Find the first derivative:

a.  $y = x^{2x}$

b.  $y = x^{\ln x}$

c.  $y = (\ln x)^x$

d.  $y = (x^2 + 1)^{4x}$

e.  $y = x^{x^2+1}$

f.  $y = (\sqrt{x})^{\sqrt{2x}}$

g.  $y = x^{e^x}$

h.  $y = (x^{x^x})$

i.  $y = (\sin x)^x$

j.  $y = x^{\cos 2x}$

k.  $y = (\tan x)^{2x}$

l.  $y = (\sin x)^{\ln x}$

## Implicit Differentiation

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### Questions:

7) Find  $y'$ :

a.  $x^2 + y^2 = 1$

b.  $x^2y^3 = x + y^2$

c.  $\frac{y^2 + x}{y^3 - 4x} = 1$

d.  $\sqrt{y} + \sqrt{x} = 1$

e.  $(y + 2)^3 = xy$

f.  $e^x + e^y = 1$

g.  $\ln x + \ln y = y$

h.  $(\ln y)^2 + y \ln x = 1$

i.  $\sin y + \cos x = y^2$

j.  $x \tan y = \sqrt{y}$

k.  $x^y + y^x = 1$

l.  $y^{\ln x} + x^{\ln y} = 4$

8) Find the first and the second derivatives:

a.  $x^2 + y^3 = 1$

b.  $x^2y^3 = x + y$

c.  $\ln x + \ln y = 1$

d.  $\sin x + \sin y = x$

## Calculations Using the Definition of Derivative

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### Questions:

9) Find the first derivative of the given function using the definition of the derivative:

- a.  $f(x) = x^2$       b.  $g(x) = x^2 + 4x + 1$       c.  $f(x) = x^3$       d.  $y = \frac{1}{x}$   
e.  $f(x) = \sqrt{x}$       f.  $z(x) = \ln x$       g.  $y = e^x$       h.  $y = \sin 2x$

## The Derivative of an Inverse of a Function

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### Questions:

- 10) Prove that  $(\ln x)' = \frac{1}{x}$ . Use the rule of derivative of the inverse function.
- 11) Prove that  $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$ . Use the rule of derivative of the inverse function.
- 12) Prove that  $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$ . Use the rule of derivative of the inverse function.
- 13) Prove that  $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$ . Use the rule of derivative of the inverse function.
- 14) Supposed that  $f^{-1}$  is the inverse function of a differentiable function  $f$  and that:  
 $f(2) = 7$ ,  $f'(2) = \sqrt{7}$ . Find  $(f^{-1})'(7)$ .

Answer Key:

1) a.  $f'(x) = 0$                       b.  $g'(x) = 0$                       c.  $h'(x) = 4x$   
 d.  $y'(x) = \frac{-2}{x^3}$                       e.  $f'(x) = \frac{1}{2\sqrt{x}}$                       f.  $z'(x) = -\frac{1}{2x^{1.5}}$   
 g.  $y'(x) = 40x^9 - \frac{1}{x^2}$                       h.  $y'(x) = 1 + 2x$                       i.  $y'(x) = 8x + 24x^2$   
 j.  $y'(x) = -\frac{2}{x^2} - e$                       k.  $y'(x) = 2\sqrt{2x} + 2e$                       l.  $y'(t) = -\frac{4}{t^2} + \frac{1}{3\sqrt[3]{t^2}}$

2) a.  $y'(x) = 3x^2 - 2x + 3$                       b.  $y'(x) = 6\sqrt{x} - 4 + \frac{5\sqrt{x}}{x}$   
 c.  $y'(x) = 3x^2 - 8x + 5$                       d.  $y'(x) = \frac{2(2x^2 + 10x - 5)}{x^2(x-1)^2}$   
 e.  $y'(x) = \frac{2(x^2 - 35 - 5)}{(3 - 2x)^2}$                       f.  $y'(x) = \frac{-2e}{(ex-1)^2}$   
 g.  $y'(x) = 12(4x+10)^2$                       h.  $y'(x) = 10x(x^2+1)^4$   
 i.  $y'(x) = \left(3x + \frac{3}{2}\right)\sqrt{x^2+x+1}$                       j.  $y'(x) = 14(2x+1)^2(4x-5)^3(2x-1)$   
 k.  $y'(x) = \frac{(2x+3)^3(2x-49)}{(x-5)^4}$                       l.  $y'(x) = -\frac{4}{3(4x+1)^{\frac{4}{3}}}$

3) a.  $y'(x) = e^x$                       b.  $y'(x) = 4e^x + 6x^2$   
 c.  $y'(x) = e^x(x^2 + 3x + 5)$                       d.  $y'(x) = \frac{e^x(x^2 - 3x + 1)}{(x^2 - x)^2}$   
 e.  $y'(x) = 4e^{4x-1} + 2e^{2x}$                       f.  $y'(x) = -xe^{-x}$   
 g.  $y'(x) = e^{-x-4}(x + e^{2x} - 1)$                       h.  $y'(x) = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$   
 i.  $y'(x) = \frac{e^{\pi x}(\pi x - (2\pi + 1))}{(x-2)^2}$                       j.  $y'(x) = \frac{-2e^{4x}}{(e^{4x} + 1)\sqrt{e^{4x} + 1}}$   
 k.  $y'(x) = \frac{2e^{x^2}x}{3\sqrt[3]{(e^{x^2} + 1)^2}}$                       l.  $y'(x) = \frac{e^{-x^2}(2x^2 + 1)}{x^2}$

4) a.  $y'(x) = \frac{1 - \ln(x)}{x^2}$ ,  $y''(x) = \frac{2 \ln(x) - 3}{x^3}$

b.  $y'(x) = \frac{2 \ln^2(x) - 1}{x \ln^2(x)}$ ,  $y''(x) = \frac{2 \ln^4(x) + 2 \ln^3(x) + \ln(x) + 2}{x^2 \ln^3(x)}$

c.  $y'(x) = 2e^{2x} \left( \ln(x^2 + 4) + \frac{x}{x^2 + 4} \right)$ ,  $y''(x) = \frac{2e^{2x} (4x^3 - x^2 + 2(x^2 + 4)^2 \ln(x^2 + 4) + 16x + 4)}{(x^2 + 4)^2}$

d.  $y'(x) = \frac{\left( \frac{1}{x} - \ln(x) \right)}{e^x}$ ,  $y''(x) = \frac{e^x (-x^2 \ln(x) - 2x - 1)}{x^2}$

5) a.  $f'(x) = \cos(x)$

b.  $g'(x) = 4 \cos(4x)$

c.  $y'(x) = -0.5 \sin(0.5x)$

d.  $y'(x) = 2 \sin(x) \cos(x)$

e.  $f'(x) = -500 \sin^3(x)$

f.  $f'(x) = \frac{\cos(2x)}{\sqrt{\sin(2x)}}$

g.  $y'(x) = \cos(x) \cos(3x) - \sin(x) \sin(3x) - 3 \sin(x)$

h.  $y'(x) = \frac{\cos(x) \cos(2x) + 2 \cos(x) + 2 \sin(x) \sin(2x) - 2 \sin(2x)}{(\cos 2x + 2)^2}$

i.  $y'(x) = 3x^2 \sin(4x) + 4x^3 \cos(4x)$

j.  $y'(x) = -\tan(x)$

k.  $y'(x) = e^{\sin 2x} \left( 2 \cos(2x) \ln(x) + \frac{1}{x} \right)$

l.  $y'(t) = -\sin(t) \cdot \cos(\cos(t))$

6) a.  $y'(x) = 2x^{2x} (\ln(x) + 1)$

b.  $y'(x) = \frac{2e^{\ln^2(x)} \ln(x)}{x}$

c.  $y'(x) = \ln(x)^x \cdot \left( \ln(\ln(x)) + \frac{1}{x} \right)$

d.  $y'(x) = 4(x^2 + 1)^{4x} \cdot \left( \ln(x^2 + 1) + \frac{2x^2}{x^2 + 1} \right)$

e.  $y'(x) = x^{x^2+1} \left( 2x \ln(x) + \frac{x^2+1}{x} \right)$

f.  $y'(x) = \sqrt{x}^{\sqrt{2x}} \cdot \frac{1}{2} \left( \frac{\ln(x)}{\sqrt{2x}} + \frac{\sqrt{2x}}{x} \right)$

g.  $y'(x) = x^{e^x} \cdot x \left( \ln(x) + \frac{1}{x} \right)$

h.  $y'(x) = x^{x^x+x} (\ln^2(x) + \ln(x) + x^{x-1})$

i.  $y'(x) = (\sin(x))^x [\ln(\sin(x)) + x \cot(x)]$

j.  $y'(x) = x^{\cos(2x)} \left[ -\sin(2x) 2 \ln(x) + \frac{1}{x} \cos(2x) \right]$



$$k. y'(x) = (\tan(x))^{2x} \left[ 2 \left( \ln(\tan(x)) + \frac{x \sin(x)}{\cos^2 x} \right) \right]$$

$$l. y'(x) = \sin(x)^{\ln(x)} \left[ \frac{\ln(\sin(x))}{x} + \ln(x) \cot(x) \right]$$

7) a.  $y'(x) = -\frac{x}{y}$

b.  $y'(x) = \frac{1-2xy^3}{3xy^2-2y}$

c.  $y'(x) = \frac{-y^3-4y^2}{-y^4-8xy-3xy^2}$

d.  $y'(x) = -\sqrt{\frac{y}{x}}$

e.  $y'(x) = \frac{y}{3(y+2)^2-x}$

f.  $y'(x) = -e^{x-y}$

g.  $y'(x) = \frac{y}{x(y-1)}$

h.  $y'(x) = -\frac{y^2}{z(2\ln y + y \ln x)}$

i.  $y'(x) = \frac{\sin x}{\cos y - 2y}$

j.  $y'(x) = \frac{\tan y}{\frac{1}{3\sqrt[3]{y^2}} - \frac{x}{\cos^2 y}}$

k.  $y'(x) = -\frac{y^x(\ln y + 1)}{x^y \left( \ln x + \frac{x}{y} \right)}$

l.  $y'(x) = -\frac{y \ln y}{x \ln x}$

8) a.  $y'(x) = \frac{-2x}{3y^2}, y''(x) = \frac{2y^2 + 8x^2}{9y^6}$

b.  $y'(x) = \frac{1-2xy^3}{3x^2y^2-1}, y''(x) = -2 \left[ y^3 + 3xy^2 \frac{1-2xy^3}{3x^2y^2-1} \right] [3x^2y^2-1] - 6(1-2xy^3) \left( xy^2 + 2x^2y \frac{1-2xy^2}{3x^2y^2-1} \right)$

c.  $y'(x) = -\frac{y}{x}, y''(x) = \frac{2y}{x^2}$

d.  $y'(x) = \frac{1-\cos(x)}{\cos(y)}, y''(x) = \frac{(1+\sin(x))\cos(y)}{\sin(y)(\cos(x)-1)}$

9) a.  $y'(x) = 2x$

b.  $y'(x) = 2x+4$

c.  $y'(x) = 3x^2$

d.  $y'(x) = \frac{-1}{x^2}$

e.  $y'(x) = \frac{1}{2\sqrt{x}}$

f.  $y'(x) = \frac{1}{x}$

g.  $y'(x) = e^x$

h.  $y'(x) = 2\cos(x)$

10-13) refer to the videos

14)  $\frac{1}{\sqrt{7}}$

## Logarithmic Differentiation

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### Questions:

- 1) Given the following function:  $y = \sqrt[4]{\frac{10x-1}{x+1}} \cdot \sqrt[10]{(2x+1)^7}$ , find  $y'$ .
- 2) Given the following function:  $y = \left(\sqrt[4]{10x+1}\right)^{2x}$ , find  $y'$ .
- 3) Find the equation of the line that is tangent to the curve:  $f(x) = x^3 - 4x^2 + 2x - 5$   
At the point on the curve where  $x = 1$ .  
Does the line intersect the curve at any other point?

### Answer Key:

- 1)  $y' = y \left( \frac{1}{4} \frac{10}{10x-1} - \frac{1}{4} \times \frac{1}{x+1} + \frac{7}{10} \frac{2}{2x+1} \right)$
- 2)  $y' = \frac{1}{4} 2^x y \left[ \ln 2 \ln(10x+1) + \frac{10}{10x+1} \right]$
- 3)  $y = -3x - 3$ , no.