

Workbook



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Surface Integrals

General Calculations with Surface Integrals

Questions:

- 1) Compute $\iint_S x^2 yz dS$ where S is the part of the plane $z = 1 + 2x + 3y$ above the rectangle $R = [0, 3] \times [0, 2]$.
- 2) Compute $\iint_S x dS$ where S is the surface $0 \leq x \leq 2$, $0 \leq z \leq 2$, $y = x^2 + 4z$.
- 3) Compute $\iint_S yz dS$ where S is the part of the plane $z = y + 3$ inside the cylinder $x^2 + y^2 = 1$.
- 4) Compute $\iint_S (x^2 z + y^{2z}) dS$, where S is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$.
- 5) Compute $\iint_S xyz dS$, where S is the part of the cone $\mathbf{r}(u, v) = u \cos v \mathbf{i} + u \sin v \mathbf{j} + 3u \mathbf{k}$ satisfying $1 \leq u \leq 2$, $0 \leq v \leq \frac{\pi}{2}$.
- 6) Compute the surface area of a sphere with radius R , $\iint_S (x^2 z + y^2 z) dS$, where S is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$.
- 7) The thin sheet S is the part of the paraboloid $z = x^2 + y^2$ below the plane $z = 1$ and it has a constant density $\delta(x, y, z) = \delta_0$. Compute the mass of the sheet.

In each of the exercises 8-12 compute $\iint_S \mathbf{F} \cdot \mathbf{n} dS$ where \mathbf{n} is the outward unit normal to S .

8) $\mathbf{F} = \left(\frac{x^2 y}{1+y^2} + 6yz^2 \right) \mathbf{i} + \arctan y \mathbf{j} - \frac{2xz(1+y) + 1 + y^2}{1+y^2} \mathbf{k};$

S is the open surface $z = 4 - x^2 - y^2, z \geq 0$.

9) $\mathbf{F} = x\mathbf{i} - 2y\mathbf{j} + 3z\mathbf{k}; S$ is the sphere $x^2 + y^2 + z^2 = 1$.

10) $\mathbf{F} = (2xy + z)\mathbf{i} + y^2\mathbf{j} - (x + 3y)\mathbf{k}; S$ is the surface of the pyramid determined by the planes: $2x + 2y + z = 6, x = 0, y = 0, z = 0$.

11) $\mathbf{F} = 5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}; S$ is the part of the paraboloid $z = 4 - x^2 - y^2$ where $z \geq 0$.

12) $\mathbf{F} = 0\mathbf{i} - 2z\mathbf{j} + (-3y - 1)\mathbf{k}; S$ is the hemisphere centered at the origin, with radius 4, and above the xy -plane.

Answer Key:

1) $171\sqrt{14}$

2) $\frac{33\sqrt{33} - 17\sqrt{17}}{6}$

3) $\frac{\pi\sqrt{2}}{4}$

4) 16π

5) $\frac{93}{\sqrt{10}}$

6) $4\pi R^2$

7) $\frac{\pi\delta_0}{6}(5\sqrt{5} - 1)$

8) -4π

9) $\frac{8\pi}{3}$

10) $37\frac{1}{2}$

11) 12π

12) -16π