proprep™

# Workbook

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# Hypothesis Testing about a Proportion

## Chances of Errors and Power

### Theory

| | Decision | | |
|---|---|---|---|
| **Real Situation** | | $H_0$ | $H_1$ |
| | $H_0$ | No error | Type I error |
| | $H_1$ | Type II error | No error |

$\alpha = P$ (rejecting $H_0$, given that $H_0$ is correct), this is the Type I error.

$\beta = P$ (accepting $H_0$, given that $H_1$ is correct), this is the Type II error.

$(1-\alpha) = P$ (accepting $H_0$ given that $H_0$ is correct), no error.

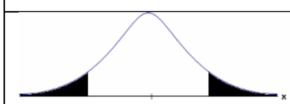
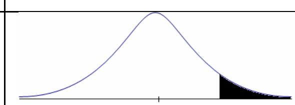
$\pi = (1-\beta) = P$ (rejecting $H_0$ given that $H_1$ is correct), no error.
This is called 'the power of the test'.

**Example** (solution in the recording)

A dentist claims that more than half of the adult population in Massachusetts does not receive the necessary regular dental treatment.
To check this claim, a survey of 150 adults is conducted.

a. List the hypotheses and decision criterion at a 10% level of significance.
b. What are the chances of a Type I error? Interpret what this means.
c. What is the test's power, if 60% of the population does not visit dentists regularly?

## The Process:

| | | | |
|---|---|---|---|
| **The Null Hypothesis:**<br><br>**The Alternative Hypothesis:** | $H_0 : p = p_0$<br><br>$H_1 : p \neq p_0$ | $H_0 : p = p_0$<br><br>$H_1 : p < p_0$ | $H_0 : p = p_0$<br><br>$H_1 : p > p_0$ |
| **Conditions:** | $np_0 \geq 10$ & $n(1-p_0) \geq 10$ | | |
| **Decision Criterion:** | $Z_{\hat{p}} < -Z_{1-\frac{\alpha}{2}}$ or $Z_{\hat{p}} > -Z_{1-\frac{\alpha}{2}}$ | $Z_{\hat{p}} < -Z_{1-\alpha}$ | $Z_{\hat{p}} > -Z_{1-\alpha}$ |
| Rejection region of $H_0$: |  |  |  |
| | $-Z_{1-\frac{\alpha}{2}}$     $Z_{1-\frac{\alpha}{2}}$ | $-Z_{1-\alpha}$ | $Z_{1-\alpha}$ |
| | ■ - Reject $H_0$ | ■ - Reject $H_0$ | ■ - Reject $H_0$ |

### The Test Statistic

$$Z_{\hat{p}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

**Example** (solution in the recording)

In January of this year, it was reported that the unemployment rate is 8%.
A current sample of 200 people was taken, with a sample unemployment rate of 6.5%.
Test at a 5% level of significance whether the unemployment rate has changed since January.

## Questions

**1)** A certain college is known to have a 25% acceptance rate.
This year, 22 of 120 candidates were accepted.
At a 5% level of significance, test whether the college has become more selective.

**2)** In a random sample of 300 voters, 171 opposed a given bill.
Would you say that a majority of voters opposes the bill?
Test at a 10% level of significance and assume that the conditions for inference are met.

**3)** A coin is tossed 50 times, and tails is obtained 28 times.
At a 5% level of significance, is the coin balanced?

**4)** A cafeteria at a certain college estimates the percentage of students buying coffee at the cafeteria at 20%.
A survey is conducted of 200 students, 33 of whom buy coffee at the cafeteria.
The purpose of the survey is to test the reliability of the cafeteria's estimate.
What conclusion would you draw at a 5% level of significance?

**5)** A Congress member wants to pass a bill. He samples 300 voters in order to test whether a majority of voters supports the bill. 276 voters in the sample support the bill.
What is your conclusion at a 5% level of significance?

**6)** The Department of Health published a statement that 10% of Americans suffer from asthma. A study seeks to check whether asthma rates are worse in Chicago because of air pollution. A random sample of 260 Chicago residents is taken.

   a. List the study hypotheses and devise a test for them at a 5% level of significance.
   b. What is the probability of a Type I error? Explain what this means.
   c. What is the power of the test in Question a, assuming that the actual asthma rate in Chicago is 16%?

**7)** The proportion of those taking a certain drug suffering from side effects is 15%.
A pharmaceutical company says it has developed a new drug that can reduce the frequency of side effects.
To test this claim, 120 patients are randomly selected to receive the new drug.
If the new drug does indeed reduce the frequency of side effects to 10%, what is the power of the test at a 5% level of significance?

**8)** 20% of the residents of a certain city are college graduates.
Following the opening of a new college, we want to check whether the proportion of university graduates has increased at a 5% level of significance.
The study includes 200 randomly selected people.
Calculate the chances of a Type II error under the assumption that the proportion of college graduates is 28%.

**9)** Is it possible to make a Type I error and a Type II error at the same time? Explain.

## Answer Key

**1)** Yes, the college has become more selective.

**2)** Yes, we accept $\mathrm{H}_a$.

**3)** Yes, the coin is fair.

**4)** We stay with $\mathrm{H}_0$.

**5)** We reject $\mathrm{H}_0$.

**6)** a. $\mathrm{H}_0 : p = 0.10, \quad \mathrm{H}_a : p > 0.10$, we reject $\mathrm{H}_0$. b. $5\%$ c. $P(Z \geq -1.2931) \approx 0.9020$

**7)** $P(Z \leq -0.1315) = 0.4477$

**8)** P(Type II error) is about $14.5\%$.

**9)** No, because a Type I error occurs when our conclusion is $\mathrm{H}_a$,
and a Type II error occurs when our conclusion is $\mathrm{H}_0$.