proprep™

Workbook

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Power Series

Region \ Radius of Convergence

Questions:

1) Find the region of convergence of the series:

a. $\sum_{n-1}^{\infty} \frac{1}{4n+1}\left(\frac{1-x}{1+x}\right)^n$

b. $\sum_{n=1}^{\infty} \frac{2^n}{n!(x-5)^n}$

c. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)10^n(x-4)^n}$

d. $\sum_{n=1}^{\infty} \frac{1}{n\ln^4 nx}$

e. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^x}$

f. $\sum_{n=1}^{\infty} \frac{1}{(x+n)(x+n-1)}$

2) Check the uniform convergence of the series:

a. $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2} \quad -\infty < x < \infty$

b. $\sum_{n=1}^{\infty} \frac{x^n}{n^{\frac{3}{2}}} \quad -1 \le x \le 1$

c. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}+x^2} \quad -\infty < x < \infty$

d. $\sum_{n=1}^{\infty} \frac{n+1}{\sqrt{n!}}\left(x^n + x^{-n}\right) \quad \frac{1}{4} \le x \le 4$

e. $\sum_{n=2}^{\infty} \ln\left(1+\frac{x^2}{n\ln^2 n}\right) \quad -a \le x \le a$

f. $\sum_{n=1}^{\infty} \frac{n^2 x}{1+n^7 x^2} \quad -\infty < x < \infty$

3) Find the range and radius of convergence of the series:

a. $\sum_{n=0}^{\infty}\frac{x^n}{n+1}$

b. $\sum_{n=0}^{\infty}\frac{(-1)x^n}{n!}$

c. $\sum_{n=1}^{\infty}\frac{5^n}{n^2}x^n$

d. $\sum_{n=1}^{\infty}x^n\sin^2\frac{1}{n}$

e. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(x+2)^n}{\sqrt{n}}$

f. $\sum_{n=1}^{\infty}\frac{(n+1)^5}{(2n+1)}x^{2n}$

g. $\sum_{n=0}^{\infty}\frac{n!}{3^n}(x-1)^n$

h. $\sum_{n=1}^{\infty}\frac{1\cdot3\cdot5\cdot.....\cdot(2n-1)}{(2n-2)!}x^n$

i. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x+1)^n}{n\cdot4^n}$

j. $\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n(x+5)^n$

k. $\sum_{n=1}^{\infty}\frac{(x-1)^{2n}}{n^4\cdot100^n}$

l. $\sum_{n=1}^{\infty}\frac{(x+5)^{2n+1}}{n\cdot2^{2n+1}}$

4) Expend the functions as a power series and find the range and radius of convergence:

a. $f(x)=\frac{1}{1+x}$

b. $f(x)=\frac{3}{1-x^4}$

c. $f(x)=\frac{1}{1+9x^2}$

d. $f(x)=\frac{1}{x-5}$

e. $f(x)=\frac{x}{4x+1}$

Answer Key:

1) a. Converges absolutely

b. Converges absolutely

c. Converges for $x \geq 4\frac{1}{10}$ or $x < 3\frac{9}{10}$

d. Converges

e. Converges for $x > 0$

f. Converges to $\frac{1}{x}$ for $x \neq 0, -1, -2, -5....$

2) a. Yes b. Yes c. Yes d. Yes e. Yes f. Yes

3) a. 1 , $-1 \leq x < 1$

b. ∞ , $-\infty < x < \infty$

c. $\frac{1}{5}$, $-\frac{1}{5} \leq x \leq \frac{1}{5}$

d. 1 , $-1 \leq x \leq 1$

e. 1 , $-3 \leq x \leq -1$

f. 1 , $-1 \leq x \leq 1$

g. 0 , $x = 1$

h. ∞ , $-\infty \leq x \leq \infty$

i. 4 , $-5 < x \leq 3$

j. $\frac{4}{3}$, $-6\frac{1}{5} \leq x \leq -3\frac{2}{3}$

k. 10 , $-9 \leq x \leq 11$

l. 2 , $-7 \leq x \leq -3$

4) a. $|x| < 1$ b. $|x| < 1$ c. $-\frac{1}{3} < x < \frac{1}{3}$ d. $-5 < x < 5$

e. $-\frac{1}{4} < x < \frac{1}{4}$