

# Workbook



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# Polar Coordinates

## Introduction

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### Questions:

- 1) Given the point with polar coordinates  $(r, \theta) = \left(3, \frac{\pi}{5}\right)$ , find three other sets of coordinates for the same point with different angles  $\theta$  in the range  $-2\pi \leq \theta \leq 2\pi$ .

## Conversion between Polar and Cartesian Coordinates

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### Questions:

- 2) Convert the following sets of Cartesian coordinates to polar coordinates:
- a.  $(1, \sqrt{3})$       b.  $(-2, -2)$       c.  $(0, -5)$       d.  $(-3, 4)$
- 3) Convert the following sets of polar coordinates to Cartesian coordinates:
- a.  $\left(\sqrt{8}, \frac{3\pi}{4}\right)$       b.  $\left(-4, \frac{2\pi}{3}\right)$       c.  $\left(0, \frac{\pi}{2}\right)$       d.  $\left(6, -\frac{\pi}{3}\right)$

## Conversion between Polar and Cartesian Coordinates

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### Questions:

- 4) Convert the following equations in  $x, y$  to polar form in  $r, \theta$ :
- a.  $2x - 5x^3 = 1 + xy$       b.  $x^2 + y^2 = 6y$       c.  $x = 3$       d.  $y = -4$

5) Convert the following polar equations in  $r, \theta$  to Cartesian form in  $x, y$ :

a.  $r = -8\cos\theta$

b.  $6r^3 \sin\theta = 4 - \cos\theta$

c.  $r = 2$

d.  $\theta = \frac{\pi}{4}$

6) Sketch the graph of  $r = 2 + 4\sin\theta$ .

## Tangent Lines in Polar Coordinates

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Questions:

7) Find the equation(s) of the tangent line(s) to  $r = \sin(4\theta)\cos(\theta)$  at  $\theta = \frac{\pi}{6}$ .

8) Find the equation(s) of the tangent line(s) to  $r = 1 - 2\sin\theta$  at the origin.

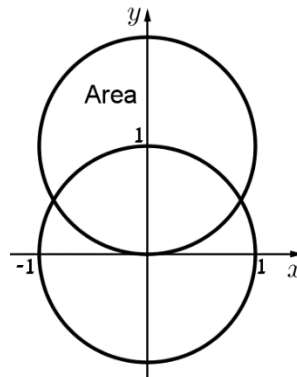
## Area in Polar Coordinates

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Questions:

9) Find the area inside the graph of  $r = 6 + 4\cos\theta$  and to the left of the  $y$ -axis.

10) Find the area between the circles  $r = 1$  and  $r = 2\sin\theta$  as in the picture:

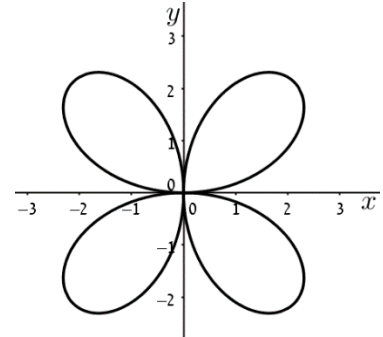


## Arc Length in Polar Coordinates

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### Questions:

- 11) Find the length of the circumference of the cardioid  $r = 1 + \cos \theta$ .
- 12) Find the arc length of one petal of the rose  $r = 6 + 4 \cos \theta$ .  
 [No need to evaluate the integral]



## Surface Area in Polar Coordinates

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### Questions:

- 13) Find the surface area obtained by revolving the curve  $r = \cos \theta$ ,  $0 \leq \theta \leq \frac{\pi}{2}$  about the  $x$ -axis
- 14) Find the surface area obtained by revolving the curve  $r = 4 + 4 \sin \theta$ ,  $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$  about the  $y$ -axis.

Answer Key:

- 1)  $\left(3, -\frac{9\pi}{5}\right)$        $\left(-3, \frac{6\pi}{5}\right)$        $\left(-3, -\frac{4\pi}{5}\right)$
- 2) a.  $\left(2, \frac{\pi}{3}\right)$       b.  $\left(2\sqrt{2}, \frac{5\pi}{4}\right)$       c.  $\left(5, \frac{3\pi}{2}\right)$       d. (5, 2.214)
- 3) a. (-2, 2)      b.  $(2, -2\sqrt{3})$       c. (0, 0)      d.  $(3, -3\sqrt{3})$
- 4) a.  $1+r^2 \cos \theta \sin \theta$       b.  $r = 6 \sin \theta$       c.  $r \cos \theta = 3$       d.  $r \sin \theta = -4$
- 5) a.  $(x+4)^2 + y^2 = 16$       b.  $4\sqrt{x^2 + y^2} - x$       c.  $x^2 + y^2 = 4$       d.  $y = x$

6) Final Graph in the side.

- 7)  $y = \frac{1}{3\sqrt{3}}x + \frac{1}{4}$
- 8)  $y\left(\theta = \frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}x$ ,  $y\left(\theta = \frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{3}x$

9)  $A = 22\pi - 48$

10)  $A = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$

11)  $L = 8$

12)  $L = 3 \int_0^{\frac{\pi}{2}} \sqrt{\sin^2 2\theta + 4 \cos^2 2\theta} d\theta$

13)  $S = \pi$

14)  $\frac{512\pi}{5}$

