

# Workbook



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# Work and Energy

## Energy Conservation and the Work Energy Theorem Part A

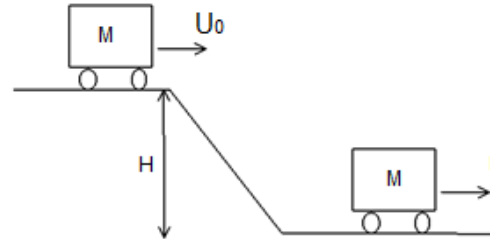
### Questions

**1) Conservation of Energy.**

A cart is moving along a frictionless surface. The cart begins above a slope of a height  $H$  with an initial velocity of  $U_0$ .

Given:  $H, U_0$ .

Find the velocity of the cart at the bottom of the slope.

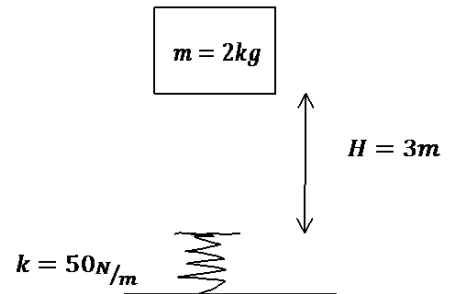


**2) An Object is dropped above a Vertical Spring.**

A massless spring with a spring constant of  $50\text{ N/m}$  is attached to the ground.

An object with a mass of  $2\text{ kg}$  is released from a stationary position  $3\text{ m}$  above the spring.

- Find the maximal compression of the spring.
- What is the maximal height the object will reach after hitting the spring?



**3) One Mass on a Slope Tied to Another Mass Hanging Vertically.**

A mass  $m_1$  is sitting on an incline with an angle  $\theta$ .

The mass is resting on a spring with a spring constant of  $k$  compressed to  $\Delta x = d$ .

There is a string tied to the mass which passes over an ideal pulley and is attached to the mass  $m_2$ , which is sitting at a height  $H$  above the ground.

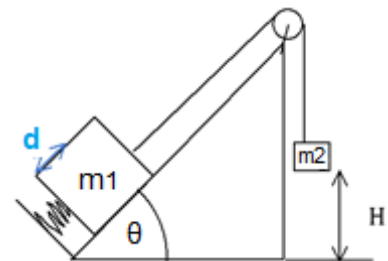
Given Values:

$$m_1 = 1\text{ kg}, m_2 = 2\text{ kg}, H = 3\text{ m},$$

$$k = 100\text{ N/m}, \theta = 30^\circ, d = 30\text{ cm}.$$

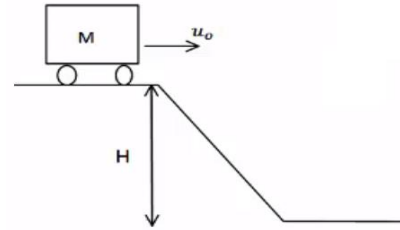
The system is released from a resting position.

Find the velocity with which  $m_2$  hits the ground.



## Energy Conservation and the Work Energy Theorem Part B

- 4) A cart is moving on a surface with friction. The cart begins above a slope with a height  $H$  and an initial velocity of  $u_0$ .  
 Given:  $u_0, H$ .  
 Find the velocity of the cart the bottom of the slope.



## Calculating Work When Force is not Constant

- 5) Calculate the Work exerted by the force  $F = x\hat{x} + yx\hat{y}$  between the points  $A(0,0)$  and  $B(2,4)$ , with the following trajectories:
- The trajectory is a straight line between the two points.
  - The trajectory is parallel to the  $x$ -axis until point  $C(2,0)$  and then parallel to  $y$ -axis until point  $B$ .
  - The trajectory is  $y = x^2$ .
  - The trajectory is  $x(t) = 2t, y(t) = 4t^2$ .

## How to check if a Force is Conservative

- 6) The force  $F$  is given:  $F = 2xy\hat{x} + (x^2 + z)\hat{y} + y\hat{z}$ .  
 Is the force  $F$  conservative?

## Calculating Potential Energy from Conservative Forces

- 7) Find the potential energy of the force:  $F = 2xy\hat{x} + (2 - x^2)\hat{y}$ ,  
 if you are given:  $U(0,0) = 0$ .

**Answer Key**

1)  $v_f = \sqrt{u_0^2 + 2gH}$

2) a.  $\Delta x = 2\text{m}$                       b. Same height it was released from.

3)  $5.745 \frac{\text{m}}{\text{sec}}$

4)  $v_f = \sqrt{u_0^2 + 2gH(1 - \mu_k \cot \theta)}$

5) a.  $W_{A \rightarrow B} = \frac{4}{2} + \frac{4 \cdot 8}{3}$               b.  $W_{A \rightarrow B} = 18$               c.  $W_{A \rightarrow B} = 2 + \frac{64}{5}$               d. Same as part c.

6) Yes.

7)  $U = x^2y - 2y$