

Workbook



Table of Contents

Math Introduction.....	2
Coordinates and Differential Elements.....	2
Density	2
Infinitesimal Density	2
Infinitesimal Calculus	3

Math Introduction

Coordinates and Differential Elements

Questions

1) Circular Area.

Calculate the area of a disk of radius R by integrating over surface elements in cylindrical coordinates (using an integral of an area element in cylindrical coordinates).

2) Calculating Cylindrical Volume.

Calculate the volume of a cylinder of height h and radius R by integrating a volume element in cylindrical coordinates.

Density

Questions

3) Disc with a Hole.

Find the density of a disc with radius R and mass M .
The disc has a hole drilled out of it with the radius r .
Find the mass that has been removed from the disc.

Infinitesimal Density

Questions

4) Stake with Non-Uniform Density.

Calculate the total mass of a stake with the length L and a mass density of $\lambda(x) = \lambda_0 \frac{x}{L}$,
when x is the distance from the left edge of the stake, and L and λ_0 are given constants.

Infinitesimal Calculus

Questions

5) Implicit Derivative.

You are given the following function: $f(x, y) = y^{\sin x} + 6y + e^{x^2+y^2} = 0$.

Solve for: $\frac{dy}{dx}$.

6) Changing Coordinates of a Linear Element.

You are given the following new coordinates: $r' = \frac{1}{r^2}, \theta' = \frac{1}{2} = \theta$.

The initial cylindrical coordinates are: r and θ .

Find the length of the length element dl as a function of the new coordinates.

Answer Key

1) πR^2

2) $\pi R^2 h$

3) $M \left(\frac{r}{R} \right)^2$

4) $\frac{\lambda_0 L}{2}$

5) $\frac{dy}{dx} = - \frac{(\ln y)(\cos x)(y^{\sin x}) + 2xe^{x^2+y^2}}{\sin x \cdot y^{(\sin x-1)} + 6 + 2ye^{(x^2+y^2)}}$

6) $dl = \sqrt{\frac{1}{4}r'^{-3} dr'^2 + \frac{1}{r'} 4d\theta'^2}$