proprep™

Workbook

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Gravity and Central Force

Motion under Gravitational Force and Central Force

Questions

1) Rocket Fired from Earth Returns.

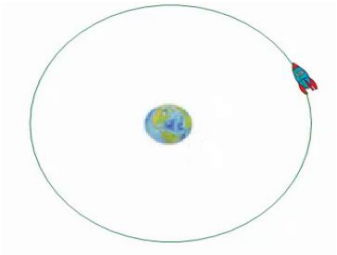
A rocket is launched from Earth. The rocket moves away from Earth and then returns. We are told that at some point in the trajectory the rocket's distance from earth is R_1. The angle between R_1 and the instantaneous velocity, v_1, is $30°$. Earth's radius is R_E and the rocket's angle of impact on Earth is $\theta°$.

a. Calculate: θ_0, V_0, V_1, V_2.
   V_2 is the rocket's impact velocity with Earth.

b. Find $R_{\max}$ and $v_{\min}$ ($R_{\max}$ is the maximum distance away from Earth which the rocket reaches, and $v_{\min}$ is the corresponding velocity).

2) Part of a Rocket at Escape Velocity.

A spaceship of mass m orbits Earth in a circular trajectory of radius R. At some point the rocket splits into two parts. One part, of mass $\frac{1}{3}m$, flies off with a velocity equal to the escape velocity in the radial direction. Find the minimum and maximum radii of the second piece.



3) Effective Potential.

A body of mass m moves in circular motion with a potential of $U(r) = -\frac{A}{\sqrt{r}}$, where A is a known constant. The angular momentum of the body is L.

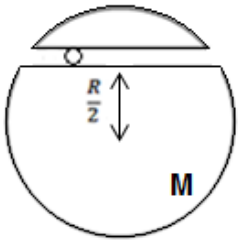
a. What is the radius of the circle?
b. What is the velocity of the body?

4) Time Period.

A body of mass m moves on a straight line, with a potential of $U(x) = B|x|$, where B is constant. The greatest distance reached by the body is A.

a. Find the general value for the energy of the body.
b. What is the period?

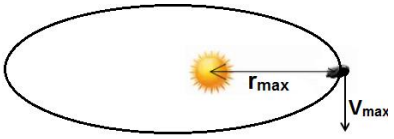
5) Body Moves Through Tunnel.



A body travels through a tunnel located $\frac{R}{2}$ from the center of a sphere of mass M.
The body begins its motion from rest at one end of the tunnel, and there is no frictional force acting.
What is the position of the body as a function of time?

Kepler's Laws

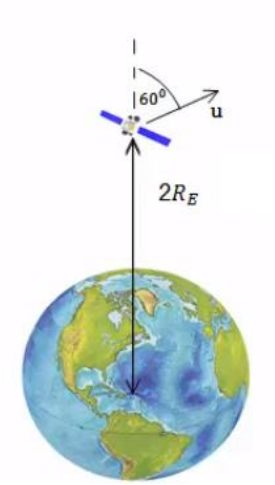
6) Finding Time Period Kepler's Second Law.



A body orbits the sun with an elliptical trajectory, such that its maximum velocity and minimum distance from the sun are given. The area of the ellipse is given.
What is the time period of the body?

Further Questions

7) Satellite Orbiting.



A satellite is shot vertically from Earth. The satellite reaches a maximum height of $2R_E$. At that moment it is given a velocity, u, at an angle of $60°$ to the vertical axis from Earth.
Ignore earth's motion and rotation.

a. Find a condition for the velocity, u, such that the satellite will orbit in a closed circuit.
b. Find another condition for u, such that the satellite will not hit Earth.

Answer Key

1) Solution in the recording.

2) Solution in the recording.

3) a. $r_0 = \left(\frac{2L^2}{ma}\right)^{\frac{2}{3}}$ b. $v = \frac{L}{m\left(\frac{2L^2}{ma}\right)^{\frac{2}{3}}}$

4) a. $E = BA$ b. $T = 8\sqrt{\frac{m}{2B}} \cdot A$

5) $x(t) = -\frac{\sqrt{3}}{2} R \cos\left(\sqrt{\frac{GM}{R^3}} t\right)$

6) $T = \frac{2s_E}{r_{\min} \cdot v_{\max}}$

7) a. $|u| < \sqrt{\frac{GM_E}{R_E}}$ b. $|u| > \sqrt{\frac{GM}{2R_E}}$