proprep™

Workbook

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Exam Questions

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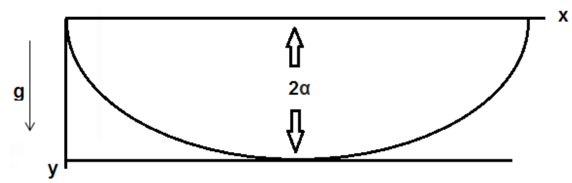
1) **Cycloid Motion.**



A point mass, m, moves along a cycloid course, as given by:

$x = \alpha(\theta - \sin\theta)$

$y = \alpha(1 - \cos\theta)$

Where α is constant and θ is the variable.

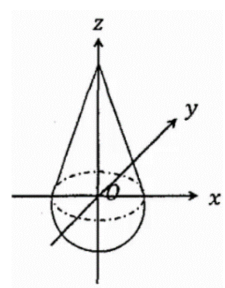
The body begins its motion from rest point $(0,0)$.

It moves through gravities field as shown in the diagram.

The point of reference for potential energy at the bottom of the trajectory, where $y = 2\alpha$.

a. What is the velocity of the mass at the lowest point of the trajectory?
b. What is the equation of motion of body through?
c. Solve the equation from b. according to the initial conditions: $y(t)$, $x(t)$, $\theta(t)$.
d. Show that the body moves in harmonic motion with a value for T (time per period), matching that of a mathematical pendulum of length l.
What is an appropriate value for l here?

2) **Center of Mass – Full Cone and Sphere.**



A body consisting of a cone, with an apex angle of $\alpha°$, a base radius of a and height h, sits on a half sphere, also of radius a. The cone and half sphere have the same uniform density of p.

a. What is the center of mass of the cone, relative to the origin, which is located on the plane connecting the cone and the half sphere?
b. If the center of mass the half sphere is located at $zCM = \frac{3}{8}a$, relative to the connecting plane from part a., what is the center of mass of the entire body?
c. The body is tilted by an angle $\theta°$ relative to the perpendicular axis.
What is the potential energy as a function of this angle?

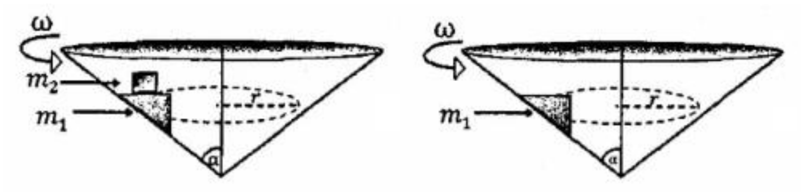
3) Masses on a Cone.

Mass m_1 is inside a cone with a half-angle of $\alpha°$, which rotates with constant angular velocity ω. The mass can move up and down the cone's wall with no friction.

a. What is the radius of rotation, r, such that mass m_1 will be at equilibrium – i.e. the mass won't move up or down the cone wall.

b. A mass, m_2, is placed on m_1, such that the coefficient of friction between them is μ_s. The rotational velocity of m_1 remains unchanged, and m_2 dose not slip when on m_1. Will the radius of motion (when the system is at equilibrium) change? Explain.

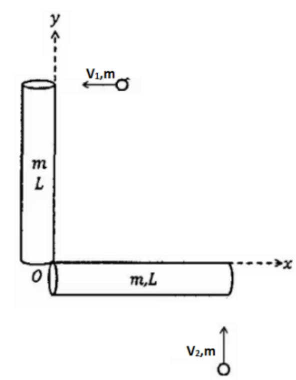
c. What is the minimum value that the coefficient of friction μ_s can be, such that there won't be slipping between the masses? The top of m_1 is horizontal.



4) Balls Hit Rods.

Two thin and long rods are at rest, each of mass m and length L. They are joined together at right angles at the origin. Two masses of mass m move perpendicularly to the rods and collide at the tip of the rods at a velocity: $\vec{v}_1 = -V_0\hat{x}$, $\vec{v}_2 = v_0\hat{y}$.

At $t = 0$ the masses stick to the rods at once.



a. What is the position vector of the center of mass $\vec{r}_{CM}(t)$, at $t = 0$?

b. What is the position vector of the center of mass $\vec{r}_{CM}(t)$, when $t > 0$, relative to the position of the center of mass at $t = 0$?
$\vec{r}_{CM}(t > 0) - \vec{r}_{CM}(t = 0) = ?$

c. What is the angular velocity, $\omega(t)$, of the system in circular motion, relative to the center of mass calculated in part b.?

d. Find the position vector $\vec{r}(t)$ of the origin, relative to the origin's position at $t = 0$.

5) Float in Harmonic Motion.

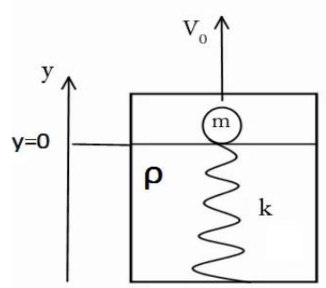
We are given a small spherical mass, m, of radius R, and an ideal, massless vertical spring, of constant k. The spring is positioned in a viscose liquid of density ρ and viscosity η.
The spring is slack when it is at the surface of the liquid, as described in the diagram.
Remember that bouyancy and stokes force are given by ρVg,
(where V is the volume of the sphere) and by $6\pi\eta R\dot{y}$.

a. When the mass is located on the surface of the liquid, it has an initial velocity of v_0 in the upwards direction.
What will be the maximum height which the mass will reach?

b. What is the mass' equation of motion when travelling through the liquid?
Assume that from the moment the mass comes in to contact with the liquid, the whole sphere is Submerged. Assume also that the face of the surface is unchanged by the entering sphere.
Hint: for ease, use variable substitution.

c. Assuming weak damping, what is the general solution for the equation of motion inside the liquid? What are the initial conditions for the motion?
The final answers must be presented in terms of the variable that was used before variable substitution was used.
Hint: when solving your differential equation, use equation sheet.

d. Once the mass has entered the liquid, how long will it take the mass to return to the surface (the state described at the start of part b.)?



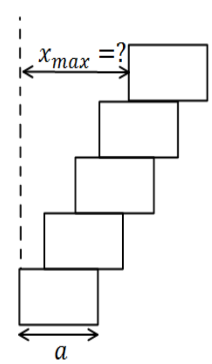
6) Tower of Bricks.

Charlie is trying to build a tower from five identical blocks, each with sides of length a.

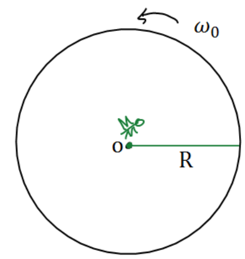
a. What is the maximum distance that he can place the uppermost block such that the tower will not fall?

b. Measure the distance between the left side of the bottom block to the left side of the tower will not fall?

c. Measure the distance between the left side of the bottom block to the left side of the top block.
Hint: begin calculating from the top block.



7) Fly on Disk.

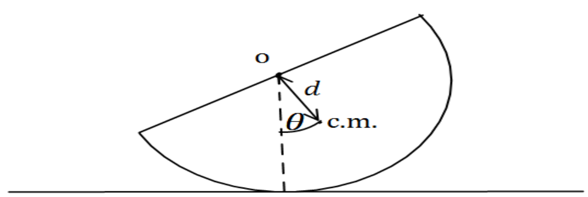


A flat disk of mass M and radius R rotates at an initial angular velocity ω_0 about its center, which is lying stationary on a frictionless table.
On the table, under the center of the disk, a green dot is drawn, O. At the center of the disk a green fly, of mass m, sleeps. A green radial line is drawn on the disk.
At $t=0$ the fly awakes and begins walking on the radial line.

a. Find the position of point O (which is on the table), relative to the fly as a function of the distance, h, between the fly and the center of the disk. Assume that the fly is at the origin, the x-axis is in the direction of the center of the disk and the y-axis is perpendicular to the x-axis, on the plane of the disk.
b. Find the angular velocity of the disk when the fly is at the edge.
c. Check your answer for part b. when:
   i) $m \ll M$;
   ii) $m \gg M$.
d. If the fly moves with a constant velocity of v_0 relative to the disk, what will be the frictional force between the fly and the disk, a moment before the fly reaches the disk's edge?=

8) Half Sphere in Harmonic Motion.

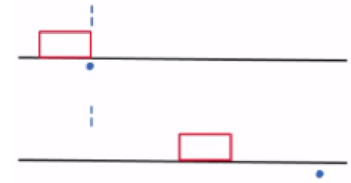
Half a sphere of radius R and mass M is resting on a surface.
The half sphere is tilted a small angle from its state of equilibrium, and is released from rest.
Find the frequency of small oscillations if the half sphere rolls without slipping (The center of mass of the half sphere is located a distance $d=\frac{3}{8}R$ from the center of a full sphere).



9) Energy Lost.

A mass, m , is placed (resting) on a conveyor belt with friction μ at a time of $t=0$.

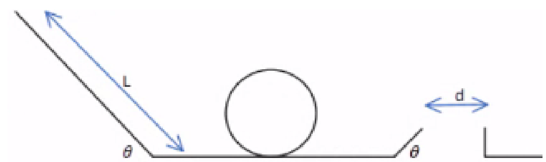
The conveyor belt is acted on by an external force which pulls the belt at a constant velocity, u .



a. What is the force acting on the conveyor belt?
b. What is the acceleration of the mass?
c. For how long will the mass continue sliding?
d. What distance will the conveyor travel in this time?
e. What distance did the mass move in this time?
f. How much work did the external force do?
g. How much work did the frictional force do?
h. How much energy was lost as heat?

10) Skateboarder.

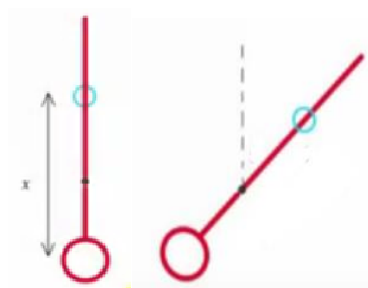
A skateboarder is skating in the skate park. The radius of the loop is R, the vertical height of the ramp is also R and the length of the jump is d .



a. What is the minimal height that L can be, in order for the skater to complete the loop?
b. What is the minimal height that L can be, in order for the skater to complete the jump?
c. We are now told that the skater can dismount the skateboard midair at a horizontal velocity of p relative to the skateboard.
   How long after the original jump must the skater begin the next jump in order to land exactly on the edge of the ditch?
d. What is the maximum distance the skater can travel safely?

11) Metronome.

Find the frequency of the metronome in the sketch, which changes according to the placement of the mass which glides along the metronome.
It is given that the axis of the metronome is located a quarter of the way up.



Answer Key

1) a. $v_f = 2\sqrt{ga}$ b+c+d. Solution in the recording.

2) a. $z_{cm} = \frac{h}{4}$ b. $z_{cm} = \frac{8h^2 - \frac{27}{2}a^2}{8h+9a}$ c. $U(\theta) = (m_1+m_2)gz_{cm}\cos\theta$

3) a. $r = \frac{g}{\omega^2 \tan\alpha}$ b. r is unchanged. c. $\mu s \geq \frac{1}{\tan\alpha}$

4) a. $\vec{r}_{cm}(t=0) = \frac{3}{8}L(1,1)$ b. $\vec{r}_{cm}(t>0) = \frac{1}{4}v_0 t(-\hat{x}+\hat{y})$ c. $\omega(t) = \frac{30}{37}\frac{v_0}{L}$

d. $\vec{r}_0 = \frac{1}{4}\left[\frac{3}{\sqrt{2}}L\cos\left(\frac{30}{37}\frac{v_0}{L}t + \frac{5\pi}{4}\right) - v_0 t\right]\hat{x} + \frac{1}{4}\left[\frac{3}{\sqrt{2}}L\sin\left(\frac{30}{37}\frac{v_0}{L}t + \frac{5\pi}{4} + v_0 t\right)\right]\hat{y}$

5) a. $h = \Delta y = \frac{-mg + \sqrt{(mg)^2 + kmv_0^2}}{k}$ b. $\ddot{z} + \frac{\lambda}{m}\dot{z} + \frac{k}{m}z = 0$

c. $z(t) = Ae^{\frac{-5}{2}t}\cos(\omega t + \phi); \quad \dot{y}(0) = -v_0, \quad y(0) = 0$

d. Solution in the recording.

6) $x_{\max} = \frac{25}{24}a$

7) a. $x_0 = \left(\frac{Mh}{m+M}\right)$ b. $\omega(R) = \frac{(M+m)^2\omega_0}{3m^2+4mM+m^2}$ c. $\omega(R)\frac{1}{3}\omega_0$

d. $f_s = -\frac{mM(m+M)^3\omega_0^2 R\hat{r}}{(3m^2+4mM+M^2)} + mMv_0\omega_0\left(\frac{2(M+m)}{3m^2+4mM+M^2} - \frac{4m}{(M+3m)^2}\right)\hat{\theta}$

8) $\omega_0 = \sqrt{\frac{15}{26}\cdot\frac{9}{R}}$

9) a. $F_{et} = \mu mg$ b. $a' = \mu g$ c. $T = \frac{u}{\mu g}$ d. $x = \frac{u^2}{\mu g}$

e. $x' = \frac{u^2}{2\mu g}$ f. $W = mu^2$ g. $W' = \frac{mu^2}{2}$ h. $\Delta E = \frac{1}{2}mu^2$

10) a. $mgL\sin\theta = \frac{1}{2}m\tilde{v}^2$ b. $v = \sqrt{gR}$ c. Solution in the recording.

d. $d_{\max} = (v_0\cos\theta + p)t$

11) $\omega^2 = \frac{-\left(m_1 g\left(x - \frac{L}{4}\right) + m_2 g\frac{L}{4} - m_3 g\frac{L}{4}\right)\theta}{m_1\left(x-\frac{L}{4}\right)^2 + m_2\left(\frac{L}{4}\right)^2 + \frac{1}{12}m_3 L^2 + m_3\left(\frac{L}{4}\right)^2}$