

Workbook



Table of Contents

Partial Derivative.....	2
Partial Derivative.....	2

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Questions

1) Compute the first order partial derivatives of the following functions:

a. $f(x, y) = 4x^3 - 3x^2y^2 + 2x + 3y$

b. $f(x, y) = x^5 \ln y$

c. (only f_x) $f(x, y) = \frac{x^2 y^4 (\sqrt{y} + 5 \ln y)}{y^2 + 5y + y^y}$

d. $f(x, y) = (x^2 + y^3) \cdot (2x + 3y)$

e. $f(x, y) = \frac{x^2 - 3y}{x + y^2}$

f. $f(x, y) = \sin(xy)$

g. $f(x, y) = \arctan(2x + 3y)$

h. $f(r, \theta) = r \cos \theta$

i. $f(x, y, z) = xy^2 z^3$

j. $f(u, v, t) = e^{uv} \sin ut$

2) Compute the second order partial derivatives of the following functions:

a. $f(x, y) = 4x^2 - x^2y^2 + 4x + 10y$

b. $f(x, y) = x^4 \ln y$

c. $f(x, y) = \sin(10x + 4y)$

d. $f(x, y, z) = xyz$

3) Answer the following questions:

a. Find the partial derivatives of the function below at the point $(0, 0)$.

b. Is the function continuous at $(0, 0)$?

c. If a function is partially differentiable at a point, must it also be continuous at that point?

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

4) Is the function from Exercise 3 differentiable at $(0, 0)$?

5) Check the differentiability of the following functions at $(0,0)$:

$$\text{a. } f(x, y) = \begin{cases} \frac{x^3 + y^3}{2x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

$$\text{b. } f(x, y) = \begin{cases} (x^2 + y^2) \sin\left(\frac{1}{\sqrt{x^2 + y^2}}\right) & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

6) Check the differentiability of the following function in its domain of definition:

$$f(x, y) = \begin{cases} e^{-\frac{1}{x^2+y^2}} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

Notation:

$f = f(x, y) \Rightarrow \begin{array}{ll} f_x = \frac{\partial f}{\partial x} = f_1 & f_y = \frac{\partial f}{\partial y} = f_2 \\ f_{xx} = \frac{\partial^2 f}{\partial x^2} = f_{11} & f_{yy} = \frac{\partial^2 f}{\partial y^2} = f_{22} \\ f_{xy} = \frac{\partial^2 f}{\partial y \partial x} = f_{12} & f_{yx} = \frac{\partial^2 f}{\partial x \partial y} = f_{21} \end{array}$

Answer Key

- 1) a. $f_x = 12x^2 - 6xy^2 + 2$, $f_y = -6x^2y + 3$ b. $f_x = 5x^4 \ln y$, $f_y = \frac{x^5}{y}$
- c. $f_x = 2x \frac{y^4(\sqrt{y} + 5 \ln y)}{y^2 + 5y + y^y}$ d. $f_x = 6x^2 + 6xy + 2y^3$, $f_y = 6xy^2 + 12y^3 + 3x^2$
- e. $f_x = \frac{x^2 + 2xy^2 + 3y}{(x + y^2)^2}$, $f_y = \frac{-3x + 3y^2 - 2x^2y}{(x + y^2)^2}$
- f. $f_x = \cos(xy) \cdot y$, $f_y = \cos(xy) \cdot x$ g. $f_x = \frac{2}{1 + (2x + 3y)^2}$, $f_y = \frac{3}{1 + (2x + 3y)^2}$
- h. $f_r = \cos \theta$, $f_\theta = -r \sin \theta$ i. $f_x = y^2 z^3$, $f_y = 2xyz^3$, $f_z = 3xy^2 z^2$
- j. $f_u = e^{uv} [v \sin ut + t \cos ut]$, $f_v = u \cdot e^{uv} \cdot \sin ut$, $f_t = u \cdot e^{uv} \cdot \cos ut$
- 2) a. $f_x = 8x - 2xy^2 + 4$, $f_{xx} = 8 - 2y^2$, $f_y = -2x^2y + 10$, $f_{yy} = -2x^2$, $f_{xy} = -4xy$, $f_{yx} = -4xy$
- b. $f_x = 4x^3 \ln y$, $f_{xx} = 12x^2 \ln y$, $f_y = \frac{x^4}{y}$, $f_{yy} = -\frac{x^4}{y^2}$, $f_{xy} = \frac{4x^3}{y}$, $f_{yx} = \frac{4x^3}{y}$
- c. $f_x = 10 \cos(10x + 4y)$, $f_{xx} = -100 \sin(10x + 4y)$, $f_y = 4 \cos(10x + 4y)$,
 $f_{yy} = -16 \sin(10x + 4y)$, $f_{xy} = -40 \sin(10x + 4y)$, $f_{yx} = -40 \sin(10x + 4y)$
- d. $f_x = yz$, $f_{xx} = 0$, $f_{xy} = z$, $f_{xz} = y$, $f_y = xz$, $f_{yx} = z$, $f_{yy} = 0$, $f_{yz} = x$, $f_z = xy$,
 $f_{zx} = y$, $f_{zy} = x$, $f_{zz} = 0$
- 3) a. The partial derivatives at the point (0,0) equal zero.
 b. The function is not continuous at (0,0).
 c. A partially differentiable function at a point is not necessarily continuous at that point.
- 4) Not differentiable.
- 5) a. Not differentiable. b. Differentiable.
- 6) Differentiable in all its domain.