

Workbook



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Questions

- 1) Given the differential equation $y' = x + y$.
 - a. Sketch line elements for the points (x, y) , where $x, y \in \{-2, -1, 0, 1, 2\}$.
 - b. Compare with a computer-aided sketch of the direction field.

- 2) Given the differential equation $y' = x + y$.
 - a. Describe the isoclines associated with this DE and sketch a few (say, 3).
 - b. Draw a few line elements on each of these.
 - c. Compare with a computer-aided sketch of the direction field.

- 3) Given the differential equation $y' = x + y$.
 - a. Show a computer-aided sketch of the associated direction field.
 - b. Use this to manually sketch the solutions to the DE passing through the points $(0, 0)$, $(0, 1)$ and $(0, -1)$.

- 4) Given the differential equation $y' = x^2 + y^2 - 2$.
 - a. Describe the isoclines associated with this DE and sketch those for slopes $c = -1, 0, 2$.
 - b. Draw a few line elements on each of these.
 - c. Compare with a computer-aided sketch of the direction field.
 - d. Use the DF to manually sketch the solutions to the DE passing through the points $(0, 0)$, $(0, 2)$ and $(0, -2)$.

- 5) Given the IVP $y' = x + y$; $y(0) = 0$.
 - a. Estimate $y(1)$ using Euler's method with step-size $h = 0.25 = \frac{1}{4}$.
[Manual computation]
 - b. Estimate $y(1)$ using Euler's method with step-size $h = 0.05 = \frac{1}{20}$.
[Computer-aided]
 - c. As above but with $h = 0.01 = \frac{1}{100}$.
 - d. Verify that $y = e^x - x - 1$ is the solution to the IVP and compute $y(1)$.
 - e. Round the results to 3 decimal places.
What can we say here about step-size and accuracy?

Answer Key

For complete solutions please refer to the videos.

