proprep™

Workbook

Table of Contents

Matrix of Linear Transformation

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Questions

1) For each of the following linear transformations, find its representation as a matrix relative to the standard basis of the appropriate $\mathbb{R}^n$:

a. $T:\mathbb{R}^2 \to \mathbb{R}^3$, $T[x,y]=[x+y,y,-x]$

b. $T:\mathbb{R}^4 \to \mathbb{R}^2$, $T[x,y,z,t]=[4x-y-z+t,x+y+4z+t]$

Remark on Notation: we'll often use row notation for convenience.

More precisely: a. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+y \\ y \\ -x \end{bmatrix}$; b. $T\begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 4x-y-z+t \\ x+y+4z+t \end{bmatrix}$.

2) Let $T:\mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by $T[x,y,z]=[4x+y-z,x-y+z]$.

Compute the representation matrix of T relative to the bases $B_1=\{(1,1,0),(0,1,1),(0,0,1)\}$ of $\mathbb{R}^3$ and $B_2=\{(1,4),(1,5)\}$ of $\mathbb{R}^2$.

I.e., find $[T]_{B_1}^{B_2}$.

Remark on Notation: we often use row notation for convenience.

More precisely: $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4x+y-z \\ x-y+z \end{bmatrix}$.

3) Find the matrix that represents the linear transformation $D:P_4[\mathbb{R}] \to P_3[\mathbb{R}]$, $D(p(x))=p'(x)$ [D for Derivative], relative to the standard bases of $P_4[\mathbb{R}]$ and $P_3[\mathbb{R}]$.

4) Find the matrix that represents the linear transformation

$$T: M_2[\mathbb{R}] \to M_2[\mathbb{R}] \quad , \quad T(A) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} A$$

relative to the basis $B = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$.

5) Let B_1 and B_2 be two bases of the space $\mathbb{R}^3$ and let T be a linear operator on $\mathbb{R}^3$.

Given that: $[M]_{B_1}^{B_2} = \begin{bmatrix} -1 & -9 & 6 \\ 1 & 6 & -4 \\ 1 & 5 & -2 \end{bmatrix}$, $\quad [T]_{B_1} = \begin{bmatrix} -29 & -45 & 6 \\ 20 & 31 & -4 \\ 13 & 19 & -1 \end{bmatrix}$,

compute: $[M]_{B_2}^{B_1}$, and $[T]_{B_2}$.

Answer Key

1) a. $[T] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 0 \end{bmatrix} = [T]_{B_1}^{B_2}$ b. $[T] = \begin{bmatrix} 4 & -1 & -1 & 1 \\ 1 & 1 & 4 & 1 \end{bmatrix} = [T]_{B_1}^{B_2}$

2) $[T] = \begin{bmatrix} 25 & 0 & -6 \\ -20 & 0 & 5 \end{bmatrix} = [T]_{B_1}^{B_2}$

3) $[D]_{E_4}^{E_3} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

4) $[T]_B = \begin{bmatrix} 1 & 2 & 2 & 0 \\ 0 & -1 & 0 & 2 \\ 3 & 5 & 4 & -2 \\ 0 & -2 & 0 & 6 \end{bmatrix}$

5) $[M]_{B_2}^{B_1} = \begin{bmatrix} 2 & 3 & 0 \\ -0.5 & -1 & 0.5 \\ -0.25 & -1 & 0.75 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & -0.5 & 1 \\ -0.75 & 2.75 & 0.5 \end{bmatrix} = [T]_{B_2}$