proprep™

Workbook

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Infinite Series

Infinite Geometric Series

Questions

1) For the following series determine if the series converges or diverges.
If the series converges give its value.

a. $\sum_{n=1}^{\infty}(0.44)^n$

b. $\sum_{n=0}^{\infty}\frac{4^n}{7^{n+1}}$

c. $\sum_{n=1}^{\infty}(-1)^n\frac{5^n}{4^{n+2}}$

d. $\sum_{n=0}^{\infty}(-4)\left(\frac{3}{4}\right)^{2n}$

e. $\sum_{n=1}^{\infty}\frac{4^n+(-5)^n}{7^n}$

f. $\sum_{n=4}^{\infty}2^{3n+4}3^{1-2n}$

g. $\sum_{n=3}^{\infty}\frac{(-5)^{3-n}}{8^{2-n}}$

h. $\sum_{n=2}^{\infty}2^{3n+4}5^{1-n}$

i. $2^{-2}+2^{-4}+2^{-6}+\ldots$

2) For the following series determine if the series converges or diverges.
If the series converges give its value.

a. $\sum_{n=1}^{\infty}\frac{1}{(n+1)(n+2)}$

b. $\frac{1}{1\cdot3}+\frac{1}{2\cdot4}+\frac{1}{3\cdot5}+\ldots$

c. $\sum_{n=1}^{\infty}\frac{1}{16n^2+8n-3}$

d. $\ln\left(1+\frac{1}{1}\right)+\ln\left(1+\frac{1}{2}\right)+\ln\left(1+\frac{1}{3}\right)+\ln\left(1+\frac{1}{4}\right)+\ldots$

Answer Key

1) a. Converges to $\frac{11}{14}=0.7857$ b. Converge to $\frac{1}{3}=0.333$ c. Diverges

d. Converge to $-\frac{64}{7}=-9.14$ e. Converge to $\frac{11}{12}$ f. Converge to $269\frac{169}{243}$

g. Diverge h. Diverges i. Converge to $\frac{1}{3}=0.333$

2) a. Converge to 0.5 b. Converge to $\frac{3}{4}$ c. Converge to $\frac{1}{12}$

d. Diverge

The Harmonic Series and the P-Series

Questions

For each of the following series determine if the series converges or diverges:

a. $\sum_{n=1}^{\infty} \frac{3}{5n}$

b. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

c. $\sum_{n=1}^{\infty} \frac{1}{n^4}$

d. $\sum_{n=1}^{\infty} \frac{1}{n^e}$

e. $\sum_{n=1}^{\infty} \frac{10}{\sqrt[3]{n^4}}$

f. $\sum_{n=10}^{\infty} n^{-2/3}$

Answer Key

a. Diverges

b. Diverges

c. Converges

d. Converges

e. Converges

f. Diverges

Algebraic Properties of Series

Questions

For each of the following series determine if the series converges or diverges:

a. $\sum_{n=1}^{\infty} \frac{10+n^2}{n^3}$

b. $\sum_{n=1}^{\infty} \frac{4n+10}{n^2}$

c. $\sum_{n=0}^{\infty} \left(\frac{4^n}{7^{n+1}} + n^{-1.5} \right)$

Answer Key

a. Diverges.

b. Diverges.

c. Converges.

The Divergence Test

Questions

For each of the following series determine if the series converges or diverges:

1) $\sum_{n=1}^{\infty} \frac{4n+5}{7n+8}$ **2)** $\sum_{n=1}^{\infty} 1$ **3)** $\sum_{n=1}^{\infty} \cos(\ln n)$

4) $\sum_{n=1}^{\infty} n$ **5)** $\sum_{n=1}^{\infty} \frac{e^n}{n^3}$ **6)** $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

Answer Key

1-6) Diverges

The Integral Test

Questions

For the following series determine if the series converges or diverges:

a. $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$ b. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+5}}$ c. $\sum_{n=1}^{\infty} ne^{-n}$

d. $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ e. $\sum_{n=2}^{\infty} \frac{1}{n\ln^3 n}$ f. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

Answer Key

a. Diverges
b. Diverges
c. Converges
d. Converges
e. Converges
f. Diverges

The Comparison Test \ The Limit Comparison Test

Questions

For the following series determine if the series converges or diverges:

a. $\sum_{n=1}^{\infty} \frac{5n^2+4n+8}{14n^5+10n^3+4n^2+10n+1}$

b. $\sum_{n=1}^{\infty} \frac{1\cdot 2\cdot 3\cdot \ldots \cdot n}{n\cdot n\cdot n\cdot \ldots \cdot n}$

c. $\sum_{n=1}^{\infty} \frac{2n^3+n^2+4n+1}{\sqrt{n^{10}+4n^4+n^2+n+1}}$

d. $\sum_{n=1}^{\infty} \frac{4n+5}{\sqrt{n^4+2n^3+n^2+4n+1}}$

e. $\sum_{n=1}^{\infty} \frac{2^n-2}{3^n+2n}$

f. $\sum_{n=1}^{\infty} \frac{5\sin^2 n}{n!}$

g. $\sum_{n=1}^{\infty} \left(\sqrt{n^2+1}-n\right)$

h. $\sum_{n=1}^{\infty} \left(1-\cos\frac{1}{n}\right)$

i. $\sum_{n=1}^{\infty} \frac{\sqrt{n}\ln n}{n^2+1}$

j. $\sum_{n=1}^{\infty} \frac{5n^2+4n+8}{14n^5+10n^3+4n^2+10n+1}$

k. $\sum_{n=1}^{\infty} \frac{1\cdot 2\cdot 3\cdot \ldots \cdot n}{n\cdot n\cdot n\cdot \ldots \cdot n}$

l. $\sum_{n=1}^{\infty} \frac{2n^3+n^2+4n+1}{\sqrt{n^{10}+4n^4+n^2+n+1}}$

m. $\sum_{n=1}^{\infty} \frac{4n+5}{\sqrt{n^4+2n^3+n^2+4n+1}}$

n. $\sum_{n=1}^{\infty} \frac{2^n-2}{3^n+2n}$

o. $\sum_{n=1}^{\infty} \frac{5\sin^2 n}{n!}$

p. $\sum_{n=1}^{\infty} \left(\sqrt{n^2+1}-n\right)$

q. $\sum_{n=1}^{\infty} \left(1-\cos\frac{1}{n}\right)$

r. $\sum_{n=1}^{\infty} \frac{\sqrt{n}\ln n}{n^2+1}$

Answer Key

a. Converges
b. Converges
c. Converges
d. Diverges
e. Converges
f. Converges
g. Diverges
h. Converges
i. Converges
j. Converges
k. Converges
l. Converges
m. Diverges
n. Converges
o. Converges
p. Diverges
q. Converges
r. Converges

The Ratio Test \ The Root Test

Questions

For the following series determine if the series converges or diverges:

a. $\sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}$
b. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$
c. $\sum_{n=1}^{\infty} \frac{(n+3)!}{n! \cdot 3^n}$
d. $\sum_{n=1}^{\infty} \frac{(2n)!}{n!(2n)^n}$
e. $\sum_{n=1}^{\infty} \frac{3^n(1+n^2)}{n!}$
f. $\sum_{n=1}^{\infty} n^{1000} e^{-n}$
g. $\sum_{n=2}^{\infty} \frac{4^n(n^2+4n+5)}{3^n \ln n}$
h. $\sum_{n=1}^{\infty} \frac{(4n^2+5n+1)^n}{4^n n^{2n}}$
i. $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

Answer Key

a. Converges
b. Converges
c. Converges
d. Converges
e. Converges
f. Converges
g. Diverges
h. Inconclusive
i. Converges

The Alternating Series Test

Questions

For the following series determine if the series converges or diverges:

a. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{4n+1}}$

b. $\sum_{n=1}^{\infty}(-1)^n \frac{\ln n}{n}$

c. $\sum_{n=1}^{\infty}(-1)^n \frac{n+2}{n^2+n}$

Answer Key

a. Converges

b. Converges

c. Converges

Absolute and Conditional Convergence of Series

Questions

1) Determine if the following series is absolutely convergent, conditionally convergent or divergent:

a. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n+1}}$

b. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{1}{n^2+1}$

c. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{\cos n}{n^2}$

d. $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n}$

e. $\sum_{n=1}^{\infty}(-1)^n \frac{\ln n}{n}$

f. $\sum_{n=1}^{\infty}\left(\frac{-1}{\ln n}\right)^n$

g. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n(n+1)}}$

2) Prove or disprove:

a. If $\sum a_n$ converges and $\sum b_n$ diverges, then $\sum(a_n+b_n)$ diverges.

b. If $\sum a_n$ diverges and $\sum b_n$ diverges, then $\sum(a_n+b_n)$ diverges.

c. 1. If $\sum(a_n)^2$ converges, then $\sum a_n$ converges.

2. If $\sum(a_n)^2$ converges and $\sum a_n$ converges, then $\sum a_n$ converges absolutely.

d. If $\sum a_n$ converges and positive, then $\sum \frac{1}{a_n}$ diverges.

e. If $\sum a_n$ converges, then $\sum(a_n)^2$ converges.

Answer Key

1) a. conditionally converges b. converges absolutely c. converges absolutely

d. conditionally converges e. conditionally converges f. converges absolutely

g. conditionally converges

2) a-e. Refer to the videos.