proprep™

Workbook

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Directional Derivatives

Directional Derivatives

Questions

1) Let $f(x,y)=x^2+y^2$.

a. Compute the gradient of f and its length at the point $(3,4)$. What is the meaning of the result?

b. Show that the gradient is normal to the contour (level curve) of f, passing through $(3,4)$.

2) Let $f(x,y)=3x^2y$.

Compute the directional derivative of f at the point $(1,2)$, in the direction of the vector $\vec{u}=3\mathbf{i}+4\mathbf{j}$.

3) Let $f(x,y)=x-\sin(xy)$.

Compute the directional derivative of f, at the point $\left(1,\frac{\pi}{2}\right)$,

in the direction of the vector $\vec{u}=\frac{1}{2}\mathbf{i}+\frac{\sqrt{3}}{2}\mathbf{j}$.

4) Let $f(x,y)=2x^2-3xy+5y^2$.

Compute the directional derivative of f at the point $(1,2)$.

in the direction of the unit vector which forms an angle of $45°$ with the positive x-axis.

5) Let $f(xy)=xy^2$.

Compute the directional derivative of f at the point $(1,3)$, in the direction of the point $(4,5)$.

6) Let $f(x,y,z)=x^2y^2z$.

Compute the directional derivative of f, at the point $(2,1,4)$, in the direction of the vector $\vec{u}=1\cdot\mathbf{i}+2\cdot\mathbf{j}+2\cdot\mathbf{k}$.

7) If the electric potential V, at the point (x, y), is given by $V = \ln\sqrt{x^2 + y^2}$, find the rate of change of the potential at the point $(3,4)$ in the direction of the point $(2,6)$.

8) Find the direction, for which the directional derivative of the function $f(x, y) = e^x(\cos y + \sin y)$, at the point $(0,0)$, is maximal, and compute its value.

9) Find the direction, for which the directional derivative of the function $f(x, y, z) = 2x^3y - 3y^2z$, at the point $(1,2,-1)$, is maximal, and compute its value.

10) If the temperature is defined by $f(x, y, z) = 3x^2 - 5y^2 + 2z^2$, and you are at the point $\left(\frac{1}{3}, \frac{1}{5}, \frac{1}{2}\right)$, in which direction should you go to cool off as quickly as possible?

Remark on Notation

a. In the plane R^2: $\mathbf{i} = \langle 1,0 \rangle$, $\mathbf{j} = \langle 0,1 \rangle$, so a vector can be denoted in two ways:
$\vec{u} = \langle x, y \rangle$ or $\vec{u} = x\mathbf{i} + y\mathbf{j}$ e.g. $\vec{u} = \langle 3,4 \rangle \Leftrightarrow \vec{u} = 3\mathbf{i} + 4\mathbf{j}$.
In 3D space R^3: $\mathbf{i} = \langle 1,0,0 \rangle$, $\mathbf{j} = \langle 0,1,0 \rangle$, $\mathbf{k} = \langle 0,0,1 \rangle$
so a vector can be denoted in two ways: $\vec{v} = \langle x, y, z \rangle$ or $\vec{v} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$
e.g. $\vec{u} = \langle 3,4,5 \rangle \Leftrightarrow \vec{u} = 3 \cdot \mathbf{i} + 4 \cdot \mathbf{j} + 5 \cdot \mathbf{k}$.

b. Elsewhere, a vector $\vec{u}$ may be denoted as $\underline{u}$ or as $\mathbf{u}$.

c. A unit vector will be denoted $\hat{\mathbf{u}}$.

Answer Key

1) The gradient is $\langle 6,8 \rangle$ and its length is 10.

2) $\frac{5}{48}$ **3)** $\frac{1}{2}$ **4)** $7.5\sqrt{2}$

5) $3\sqrt{13}$ **6)** $\frac{3}{88}$ **7)** $\frac{1}{5\sqrt{5}}$

8) It is maximal the in the direction of the vector $\langle 1,1 \rangle$ and is equal to $\sqrt{2}$.

9) It is maximal the in the direction of the vector $\langle 12,14,-12 \rangle$ and is equal to 22.

10) In the direction of the vector $\langle -2,2,-2 \rangle$.