proprep™

Workbook

Table of Contents

Applications of the Definite Integral - Volume and Surface Area

Volume – Solids of Revolution

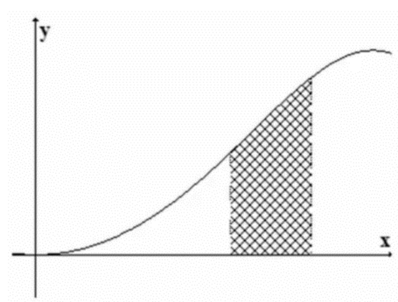
Questions

1) An area, bounded by the graphs of the function $y = x^2$ and $y = 2x$, revolves around the x-axis.
Compute the volume of the obtained solid in two ways:
   a. The disc method (due to Cavalieri).
   b. The cylindrical shell method.

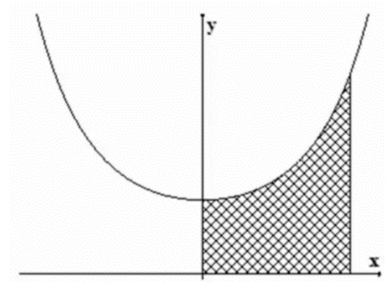
2) An area, bounded by the graphs of the function $y = x^2$ and $y = 2x$, revolves around the y-axis.
Compute the volume of the obtained solid in two ways:
   a. The disc method (due to Cavalieri).
   b. The cylindrical shell method.

3) An area, bounded by the graph of $f(x) = 1 - x^3$ and the axes, is shown in the figure.
Compute the volume of the solids obtained by revolving the area about each of the following lines:
   a. The x-axis.
   b. The line $y = -1$.
   c. The line $y = 2$.
   d. The y-axis.
   e. The line $x = -1$.
   f. The line $x = 2$.

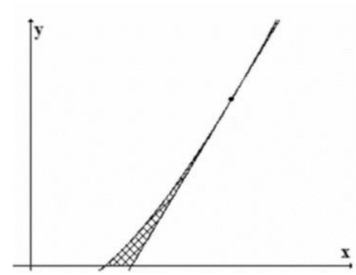
4) The area bounded by the graph of $y = \sin(x^2)$ and the lines $x = \sqrt{\frac{\pi}{6}}$, $x = \sqrt{\frac{\pi}{3}}$ and $y = 0$ is shown in the figure. What is the volume of the solid obtained by revolving it around the y-axis?



5) The area bounded by the graph of $y = e^{x^2}$ and the line $x = 0$, $x = 1$, and $y = 0$ is shown in the figure. What is the volume of the solid obtained by revolving it around the y-axis?



6) The shaded area in the figure is bounded by the graph of $y = e^{x^2}$, the tangent at (e, e) and the x-axis.
What is the volume of the solid obtained by revolving this area about the x-axis?



7) State and prove the formula for computing the volume of a cylinder.

8) State and prove the formula for computing the volume of a ball.

9) State and prove the formula for computing the volume of a cone.

Answer Key

1) $\frac{64\pi}{15}$ **2)** $\frac{8\pi}{3}$ **3)** a. $\frac{9\pi}{14}$, b. $\frac{15\pi}{7}$, c. $\frac{33\pi}{14}$, d. $\frac{3\pi}{5}$, e. $\frac{21\pi}{10}$, f. $\frac{12\pi}{5}$

4) $\frac{\pi}{2}\left(\sqrt{3}-1\right)$ **5)** $\pi(e-1)$ **6)** $\frac{\pi}{54}\left(e^3-4\right)$

7) $V = \pi r^2 h$ **8)** $V = \frac{4}{3}\pi r^3$ **9)** $V = \frac{1}{3}\pi r^2 h$

Surface Area – Solid of Revolution

Questions

1) Write the formulae for computing the surface area of the solid obtained by rotating a curve respectively about:

a. The x-axis

b. The y-axis

2) Determine the surface area of the solid obtained by rotating the graph of $y=\sqrt{4-x^2},\ -1\le x\le 1$ about the x-axis.

3) State and prove the formula for computing the surface area of a cone.

4) Determine the surface area of the solid obtained by rotating the graph of $x=\sqrt{9-y^2},\ -2\le x\le 2$ about the y-axis.

Answer Key

1) a. $S=2\pi\int_a^b f(x)\sqrt{1+\left(f'(x)\right)}dx$ b. $S=2\pi\int_c^d g(y)\sqrt{1+\left(g'(y)\right)^2}dy$

2) 8π

3) $S=\pi rl=\pi r\sqrt{h^2+r^2}$

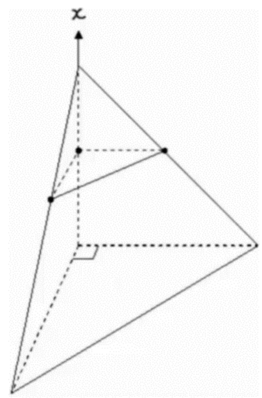
4) 24π

Volume by Integrating Cross Sections

Questions

1) Find the formula for computing the volume of a right pyramid with height h and whose base is a square with side of length a.

2) Find a formula for computing the volume of a right pyramid with height c and whose base is a right triangle with legs of lengths a and b respectively.
[For simplicity, you may assume that the apex is directly above the right-angle of the base]



Answer Key

1) $V = \frac{a^2 h}{3}$

2) $V = \frac{abc}{6}$