proprep™

Workbook

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Tangent, Normal Lines and Linear Approximation

Tangent and Normal Lines – Basic Exercises

Questions:

1) Find the equation of the line that is tangent to the curve $f(x)=x^2-2x+3$ at the point on the curve, where $x=2$.

2) Find the equation of the line that is normal to the curve $f(x)=x^2-2x+3$ at the point on the curve, where $x=2$.

3) Find the equation of the line that is tangent to the curve $f(x)=x^3-4x+2x-5$ at the point on the curve, where $x=1$, Does the line intersect the curve at any other point?

4) Find the equation of the line that is tangent to the curve $f(x)=x^3-6x+2$ and are parallel to the line $y=6x-2$.

5) Find the equation of the line that is tangent to the curve $f(x)=y=\sqrt{2x+3}$ at the point on the curve, where $x=3$.

6) Find the equation of the line that is tangent to the curve $f(x)=xe^{x^2}$ at the origin. Does the line intersect the curve at any other point?

7) Find the equation of the tangent to the curve $f(x)=e^{\sin(4x)}$ at the point on the curve, where $x=\pi$.

8) Find the equation of the lines that are normal to the curve $f(x)=\frac{2x}{1-x}$ and parallel to the line $y=-2x$.

9) Find the equation of the line that is tangent to the curve $f(x)=x\ln x$ at the point on the curve, where $x=e$.

10) Find the equation of the line that is tangent to the curve $f\left(x\right)=e^{x-1}\ln x$ at the point on the curve, where $x=1$.

11) Find the equation of the tangent to the curve $f\left(x\right)=\sin 2x$ at the point on the curve, where $x=\frac{\pi}{2}$.

12) At how many different values of x does the curve $y=x^3-2x+1$ have a tangent line parallel to the line $y=x+1$?

13) Find the equation of the tangent to $f\left(x\right)=x^{4x}$ at the point where $x=1$.

14) Find the equation of the tangent to $f\left(x\right)=\left(2x+1\right)^{x^2+1}$ at the point where $x=0$.

Answer Key:

1) $y=2x-1$

2) $y=-\frac{1}{2}x+4$

3) $y=-3x-3$, Intersections: $\left(1,-6\right),\left(2,-9\right)$

4) $y=6x-14$; $y=6x+18$

5) $y=\frac{1}{3}x+4$

6) $y=x$, No additional intersection points

7) $y=4x-4\pi+1$

8) $y=-2x-1$; $y=-2x+3$

9) $y=2x-e$

10) $y=x-1$

11) $y=-2x+\pi$

12) 2 Points

13) $y=4x-3$

14) $y=2x+1$

Tangent and Normal Lines – Exercises with a Constant

Questions:

1) Determine the constant k such that the line $y=-x+3$ is tangent to the curve $y=\frac{k}{x+1}$.

2) Determine the constant n such that the line $y=-x+n$ is tangent to the curve $y=\frac{1}{x}$.

3) Determine the constants a , c such that the line $y=ax+0.5$ is tangent to the curve $y=\frac{2}{x+c}$ at the point where $x=0$.

4) Write the equation of the straight line passing through the point $(-1,0)$ and tangent to the curve $f(x)=\sqrt{x}$. Sketch the curve and the line.

5) Write the equation of the straight lines passing through $(1.5,0)$ and tangent to the curve $f(x)=\frac{1}{4}x+1$, Sketch the curve and the lines and prove that the lines are perpendicular.

6) Find all lines that can be drawn tangent to the curve $f(x)=x^2-2x+1$ from the point $(2,-3)$. Sketch the curve and the lines.

7) Determine the constant b such that the line $y=3x$ is tangent to the curve $y=x\sqrt{x}+b$.

8) Determine the constant c, such that the curve $y=-0.5x^2+c$ is tangent to the curve $y=\frac{1}{x}$. Find the tangent point and the joint tangent.

9) Determine the constant c, such that the parabola $y=-x^2+c$ is tangent to the parabola $y=x^2-4x+6$. Find the tangent point and the joint tangent.

10) Determine the constant c, such that the curve $y=-8x^2+c$ is tangent to the curve $y=\frac{1+6x^2}{2x^2}$. Find the tangent point(s) and the joint tangents(s).

Answer Key:

1) $k=4$.

2) $n=\pm 2$.

3) $a=-\frac{1}{8}, c=5$.

4) $y=\frac{1}{2}x+\frac{1}{2}$.

5) $y=2x-3, y=-\frac{1}{2}x+\frac{3}{4}$.

6) $y=6x-15, y=-2x+1$.

7) $b=4$.

8) $c=1.5$, tangent point: $(1,1)$, joint tangent: $y=x+2$.

9) $c=4$, tangent point: $(1,3)$, joint tangent: $y=-2x+5$.

10) $c=7$, tangent point: $\left(\frac{1}{2},5\right)$, joint tangent: $y=-8x+9$

$c=7$, tangent point: $\left(-\frac{1}{2},5\right)$, joint tangent: $y=8x+9$.

Tangent and Normal Lines of Implicit Functions

Questions:

1) Find the equation of the line tangent to $x^2 + y^2 = 2xy - 2x - y + 6$, at $(2,2)$.

2) Find the equation of the line tangent to $x^3 + 4xy - 4y^3 = 1$, at the point (x, y) on the curve, where $x = 1, y > 0$.

3) Find the equation of the line tangent to $\sqrt{x} + \sqrt{y} = 2\sqrt{a},\ a > 0$, at the point on the curve, where $x = a$.

Answer Key:

1) $y = -2x + 6$

2) $y = -\frac{1}{11}x - \frac{10}{11}$

3) $y = -x + 2a$

Tangent and Normal Lines – Parametric Functions

Questions:

1) Give the following function $x = f(t)$, $y = g(t)$ where: $x = \sqrt{2t^2 + 1}$, $y = 4t^2 + 4t + 1$:

a. Find $\frac{dy}{dx}$.

b. Find the equation of the line tangent to the curve at the point for which $t = 2$.

2) Give the following function $x = f(t)$, $y = g(t)$ where: $x = \sqrt{2t^3 + 5t^2}$, $y = \sqrt[3]{4t}$:

a. Find $\frac{dy}{dx}$.

b. Find the equation of the line tangent to the curve at the point for which $t = 2$.

Answer Key:

1) a. $\frac{dy}{dx} = \frac{(4t+2)\left(\sqrt{2t^2+1}\right)}{t}$ b. $y = 15x - 20$.

2) a. $\frac{dy}{dx} = \frac{4\sqrt{2t^3+4t^2}}{3\sqrt[3]{(4t)^2 \cdot (3t^2+5t)}}$ b. $y = \frac{2}{\sqrt{22}}x - \frac{22-6\sqrt{22}}{11}$.

The Angle Between Two Curves

Questions:

1) Show that the curve $x^2 + 2y^2 = 8$ and the curve $x^2 - y^2 = 2$ intersect at right angles.

2) Find the angles between the following pairs of curves:

a. $y = x^2,\ y = \frac{1}{x}$ b. $x^2 + y^2 = 8,\ \ y^2 = 2x$

Answer Key:

1) Slope 1: $\frac{-1}{\sqrt{2}}$, Slope 2: $\sqrt{2}$ $\frac{-1}{\sqrt{2}} \cdot \sqrt{2} = -1 \Rightarrow$ The curves are perpendicular

2) a. $\alpha = 71.57°$ b. $\alpha = 71.57°$

Vertical Tangents and Cusps

Questions:

1) Answer the following questions:

a. Find all the points on the graph $y=\sqrt[5]{4-x}$ where the tangent line is vertical.

b. Does the function have a vertical cusp?

2) Answer the following questions:

a. Find all the points on the graph $y=\sqrt{x}+\sqrt[3]{x}$ where the tangent line is vertical.

b. Does the function have a vertical cusp?

3) Answer the following questions:

a. Find all the points on the graph $y=\sqrt[3]{x^2}$ where the tangent line is vertical.

b. Does the function have a vertical cusp?

4) Answer the following questions:

a. Find all the points on the graph $y=\left|x^3-27\right|$ where the tangent line is vertical

b. Does the function have a vertical cusp?

5) Answer the following questions:

a. Find all the points on the graph $y=\sqrt{4-x^2}$ where the tangent line is vertical.

b. Does the function have a vertical cusp?

6) Answer the following questions:

a. Find all the points on the graph $f(x)=\begin{cases} x^{\frac{1}{3}}+4 & x\le 0 \\ 4-x^{\frac{1}{5}} & x>0 \end{cases}$,

where the tangent line is vertical.

b. Does the function have a vertical cusp?

Answer Key:

1) a. $(4,0)$ b. No

2) a. $(2,0),(-2,0)$ b. No

3) a. $(0,0)$ b. Yes

4) a. None b. No

5) a. $(-2,0),(2,0)$ b. No

6) a. $(0,4)$ b. Yes

Linear Approximation

Questions:

1) Use linear approximation to approximate the value of $\sqrt[5]{33}$.

2) Use linear approximation to approximate the value of $\sqrt[4]{15}$.

3) Use linear approximation to approximate the value of $\sin 3°$.

4) Use linear approximation to approximate the value of $\arctan 0.25$.

5) Use linear approximation to approximate the value of $\frac{1}{e}$.

Answer Key:

1) 2.0125 **2)** 1.96875 **3)** 0.05236 **4)** 0.25 **5)** 0